Student handout

Find the Fourier Transform of the delta function.

1 Instructor's Guide

1.1 Introduction

If students know about the Dirac delta function, this is a great first example of the Fourier transform that students can work out in-class for themselves.

Students will need a short lecture giving the definition of the Fourier Transform

$$\mathcal{F}(f) = \tilde{f}(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-ikx} f(x)\, dx \tag{1}$$

We strongly suggest the convention of putting the exponential to the left of the function in the integrand to highlight the relationship between the Fourier Transform and the quantum mechanics notion of finding the projection of a quantum wavefunciton $f(x)$ onto a plane wave (complete with the complex conjugate of the plane wave that turns the ket into a bra).

We always use the convention of putting a factor of $\frac{1}{\sqrt{2\pi}}$ into the definitions of each of the Fourier Transform and its inverse to make the operations symmetric in this way. Warn students that this convention is NOT universal. They should use caution when using other resources.

1.2 Student Conversations

Students may ask where the peak of the delta function should be. If so, this is a great opportunity to (again) highlight the advantages of choosing a parameter for an unknown value so that you are doing many cases at once. Use the case $x_0 = 0$ as a limiting case.

1.3 Wrap-up

This example highlights the inherent complex number nature of Fourier transforms. Because of the factor e^{-ikx}, the Fourier transform of a real function is typically NOT real.

The answer here is trying to be the constant number $\frac{1}{\sqrt{2\pi}}$, but this number is augmented by a complex phase e^{-ikx_0} that depends on x_0m where the peak of the delta function is.

You might want to ask students to act out the Fourier transform of the delta function using the arms representation.

This example is (almost) the inverse of Fourier Transform of a Plane Wave. If you really want the inverse problem, change one of the prompts to "Find the inverse Fourier transform of ..."