Student handout Suppose you have a definite function $f(x)$ in mind and you already know its Fourier transform, i.e. you know how to do the integral

$$\tilde{f}(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-ikx}\, f(x)\, dx \tag{1}$$

Find the Fourier transform of the shifted function $f(x - x_0)$.

1 Instructor's Guide

1.1 Introduction

Students will need a short lecture giving the definition of the Fourier Transform

$$\mathcal{F}(f) = \tilde{f}(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-ikx}\, f(x)\, dx \tag{2}$$

1.2 Student Conversations

This example will feel very abstract to some students. It may be difficult for them to understand that the conditions of the problem state that the know both $f(x)$ and $\tilde{f}(k)$. This problem is about changing f slightly (by shifting its argument by x_0) and then asking how $\tilde{f}$ changes, in response.

1.3 Wrap-up

The result from this calculation underlies why it is possible to factor out the time dependence in the Fourier transform of a plane wave, Fourier Transform of a Plane Wave. Even though the problem is somewhat abstract, it is super important in applications for this reason.