Student handout Take-home messages from today's interactive lecture:

- You can think about the differential dx as a small amount of x. While technically, it is an infinitesimally small amount, in practice it is almost always possible to think of it as just really, really small. By which, we mean small enough that any error you make by giving it a finite rather than infinitesmal size is too small to make a significant difference in your calculation. Much more about this in later activities.
- A differentials equation tells you how a small change in one variable is related to a small change in one or more other variables. You can interpret these equations geometrically with a figure.
- When doing algebra with differentials equations, you can think of each differential as a new variable.
- When you zap an equation with d, the resulting differentials equation is *linear* in the differentials, i.e. each of the differentials appears to the first power. This is an open invitation to do linear algebra to rearrange the equation.
- The dimensions of dx are the same as the dimensions of x. After all, dx is just a small amount of x.
- Every equation involving differentials should have matching factors of smallness in every term. For example:

$$dx = 2y^2\,dy \qquad \text{one factor of } d \text{ in every term} \tag{1}$$

$$dx\,dy = r\,dr\,d\theta \qquad \text{two factors of } d \text{ in every term} \tag{2}$$

as opposed to:

$$\cancel{dx = 2y\,dy\,dz}$$

1 Instructor's Guide

1.1 Introduction

The vector differential $d\vec{r}$ is a fundamental unifying feature of our approach to vector calculus and therefore to electrostatics and magnetostatics. It is used both to define derivatives like the gradient and directional derivatives and integrals in space, from integrals along curves to surfaces and volumes.

This interactive lecture is the first step in defining and evaluating the vector differential. It should introduce the scalar differential and how to calculate it by "zapping with d", i.e. it is mainly procedural. Emphasize that zapping with d linearizes the equations so that it is easy to do substitution with differentials. Also emphasize the role of systems of equations.

An important non-procedural, geometric message is how the differentials (as opposed to differential) equation tells you how small changes in one variable are related to small changes in other variables.

The important takehome messages are listed in the summary handout for students.

You might want to have the student's build their understanding through a thoughtful series of SWBQ's (one at a time with class discussion after each one!)

Global prompt: "Zap the following expression with d. Then draw a figure that shows you what the differentials equation is telling you about the relationship of small changes."

- *After* they have done the following problem, remind them that most of them already know how to do this from u-substitution so that their knowledge about that is in their working memory.

$$u = y^3$$

- Also a u-substitution problem, but they need to do chain rule. Mention that this is a great time for them to be reviewing basic calculus if they need to. Also, they must know, in any give expression, which letters are constants and which are variables.

$$u = Ae^{-(\frac{x}{a})^2}$$

- This example shows that you don't have to have an equation with an isolated variable on one side AND you can have more than two variables. Look at this example both for $r =$ constant and r as a variable. Draw the figure for the case $r =$ constant. How much harder is the figure if r is variable?

$$x^2 + y^2 = r^2$$

- A system of equations. Ask the students to solve for dx and dy OR for dr and $d\phi$. Which is easy and which is hard? Remind students that in u-substitution they probably know that they need to change ALL instances of the old variable to u's, the same applies here. If they are changing to r and ϕ from x and y, they should, if possible, change ALL of the instances. Sometimes this is algebraically impossible to do, but they should at least keep trackin their minds that, if they still have x's and y's, they should now think of these as functions of r and ϕ and not independent, i.e. $x = x(r, \phi)$ and $y = y(r, \phi)$ even if they can't write down explicitly what those functions are. These nuances are particularly important in thermodynamics. (Note: We like to use ϕ for polar coordinates so that these coordinates agree with the physicists' use of spherical coordinates.)

$$r^2 = x^2 + y^2 \tag{3}$$
$$\tan\phi = \frac{y}{x} \tag{4}$$

- (Optional:) Another system of equations. Ask the students to solve for dx and dy OR for dr and $d\phi$. Which is easy and which is hard?

$$x = r\cos\phi \tag{5}$$
$$y = r\sin\phi \tag{6}$$