Use a small whiteboard to represent the tangent plane to the graph of a function of 2 variables. A small change in position in the domain corresponds to a diagonal line in the tangent plane, representing df. To get from one end to the other, one can first proceed a distance dx in the x direction, then a distance dy in the y direction, with corresponding changes to f. This triangle can be drawn on the whiteboard. These changes can be represented as the slope in the corresponding direction, multiplied by the distance, that is, as $\frac{\partial f}{\partial x}dx$, and similarly for y. The change df in f is of course the sum of these two changes in the coordinate directions, leading to the multivariable chain rule in differential form, namely

$$df = \frac{\partial f}{\partial x}\,dx + \frac{\partial f}{\partial y}\,dy$$