Prerequisites: Students should

1. know how to represent a complex number and a quantum state using the Arms representation.
2. know how to time evolve a quantum state with a time-independent Hamiltonian.
3. know that an overall phase doesn't change the quantum state.
4. Be familiar with the Hamiltonian and eigenstates for a particle confined to a ring.

Students' Task & Conversations:

1. SetUp
   a) Ask for 8 volunteers for a kinesthetic activity with arms.
   b) Ask the students to form a ring and assign each student an angular position on the ring. Tip: write the positions on name tags so that the students don't have to remember which position they correspond to.
   c) Ask the students to unwrap from the ring and stand in a line.

   a) $m = 1$ Eigenstate
   b) Write the equation for the energy eigenstate wavefunction of the ring on the board:

$$E_m(\phi) = \frac{1}{\sqrt{2\pi}} e^{im\phi}$$

   and ask the students to represent the $m = 1$ eigenstate.
      i. Point out that the norm of this wavefunction is the same for every value of ϕ, so at the arms should represent the same norm. The only different is the phase.
      ii. Point out that the phase of each arms corresponds to the position for this state. There is 1 complete spiral for this eigenstate.
   c) Ask the students to time evolve the state. Tip: You can write the equation on the board if it helps:

$$E_m(\phi, t) = \frac{1}{\sqrt{2\pi}} e^{-iE_m t/\hbar} e^{im\phi}$$

      i. Students' arms should rotate counter clockwise because of the negative sign in the time-dependent phase. If the state with the $m = -1$ state, their arms should rotate the other way.
      ii. Students' arms should all be rotating at the same rate because they are a single energy eigenstate.
      iii. How fast should the arms be rotating? Slow - this is the lowest energy.
      iv. Is the state changing? Point out that the relative phases of the individual people are not changing therefore the state is not changing. All probabilities calculated with the state will not depend on time.

2. $m = 2$ Eigenstate

   a) Ask the students to represent the $m = 2$ eigenstate.

      i. Now each phase is twice the position of each arm. There are 2 complete spirals for this eigenstate.

   b) Ask the students to time evolve the $m = 2$ state.

      i. Students' arms should still rotate counter clockwise because of the negative sign in the time-dependent phase.

      ii. Students' arms should all be rotating at the same rate because they are still a single energy eigenstate.

      iii. How fast should the arms be rotating? The energy goes like m^2:

      $$E_m = \frac{m^2\hbar^2}{2I}$$

      so 4 times as fast.

      iv. Is the state changing? Point out that the relative phases of the individual people are still not changing therefore the state is not changing. All probabilities calculated with the state will not depend on time.

3. Superposition

   a) Ask students to begin to represent a superposition state by each person representing the $m = 1$ with their left arm and the $m = 2$ state with their right arm.

      i. Tip: it could help for the each student to hold up 1 finger with their left arm and 2 fingers with their right arm to help them keep track.

      ii. Point out that really, each person should only be using 1 arm that would be a weighted vector sum of the two arms, but we're going to keep the arms seperate for now.

   b) Ask the students to time evolve each arm.

   $$\psi(\phi, t) = \frac{1}{\sqrt{2\pi}} e^{-iE_1 t/\hbar} e^{i\phi} + \frac{1}{\sqrt{2\pi}} e^{-iE_2 t/\hbar} e^{i2\phi}$$

      i. The left and right arms should be moving at different rates, with the right arm moving 4 times faster! This is really hard.

      ii. Since each person should really be 1 arm, the some of the arms would be changing with time - both the norm and the phase! For example, when 1 persons arms are poitning in opposite directions, the sum of their arms is zero, and each person will be zero at different times.

      iii. This state changes with time! This is a non-stationary state.