Students should already be familiar with:

1. Inner product for quantum states means:
   - take the Hermitian adjoint (i.e., complex conjugate the components) of one of the vectors,
   - find the product between the same component of each vector,
   - add the component-products together
2. Adding vectors tip-to-tail.
3. Multiplying two complex numbers with exponential form/Arms means multiplying arm lengths and adding phase angles

Prompts:

1. Ask a pair of students to represent an arbitrary spin state $|\psi_{student}\rangle$.
2. Have the instructor and the TA represent the state $e^{i\pi/2}|\psi_{student}\rangle$ (to avoid algebraic solutions, don't write the state on the board)
3. Ask the class: **"Are the two states orthogonal?"**
   a) If you've already discussed how the overall phase of the vector doesn't affect the quantum state, students might suggest (correctly) that they can't be orthogonal states because they are the same state!

      However, I recommend doing this activity before you talk about relative and overall phase, if possible.

   b) An appealing distractor is that the component pairs across the states (the spin up components or the spin down components) **look** perpendicular - the arms are perpendicular in real 3D space. However, the state vectors are **not** orthogonal in the Hilbert (spin state) space.

   c) To really know if two states are perpendicular to each other, the inner product between the two vector must be zero.
4. Lead a class discussion about the procedure for doing the inner product:
   a) Decide which order you want the vectors in the inner product

      (changing the order complex conjugates the result, but we're looking for zero or non-zero).

   b) Complex conjugate the first vector

      (reflect arm of both people across the horizontal/real axis)

   c) Pair the components across the two vector

      (spin-up with spin up and spin down with spin down)

   d) Multiply like components

      (multiple arm lengths and add phase angle)

      A third person could represent the product for each like-component pair

e) Add all the like-component products.

(tip to tail - put one persons shoulder at the hand of the other person. Helps if someone kneels or crouches)

For this pair of vectors, they should not get zero.

Possible FollowUps:

1. Discuss how this procedure shows that $|+\rangle_x$ and $|-\rangle_x$ are orthongal.
2. Try to discern a pattern for identifying two spins states represented with arms as orthogonal

   (if the norm of each component is the same, i.e., all arms are the same length, then the relative relative phase between the two vectors should be different by $e^{i\pi}$, i.e., spin-down arms are different by 180°).
3. Discuss how this inner product procedure generalizes to higher-order spin states (spin-1, spin-3/2, etc.) - just more component pairs to multiply together.
4. Discuss how this inner product procedure matches the algebraic notations -