1 Isothermal/Adiabatic Compressibility

The isothermal compressibility is defined as

$$K_T = -\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_T \tag{1}$$

K_T is be found by measuring the fractional change in volume when the the pressure is slightly changed with the temperature held constant. In contrast, the adiabatic compressibility is defined as

$$K_S = -\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_S \tag{2}$$

and is measured by making a slight change in pressure without allowing for any heat transfer. This is the compressibility, for instance, that would directly affect the speed of sound. Show that

$$\frac{K_T}{K_S} = \frac{C_p}{C_V} \tag{3}$$

Where the heat capacities at constant pressure and volume are given by

$$C_p = T\left(\frac{\partial S}{\partial T}\right)_p \tag{4}$$

$$C_V = T\left(\frac{\partial S}{\partial T}\right)_V \tag{5}$$