1 Completeness Relation Change of Basis

(a) Given the polar basis kets written as a superposition of Cartesian kets

$$\begin{aligned} |\hat{s}\rangle &= \cos\phi\,|\hat{x}\rangle + \sin\phi\,|\hat{y}\rangle \\ \left|\hat{\phi}\right\rangle &= -\sin\phi\,|\hat{x}\rangle + \cos\phi\,|\hat{y}\rangle \end{aligned}$$

Find the following quantities:

$$\langle\hat{x}|\hat{s}\rangle\,, \quad \langle\hat{y}|\hat{s}\rangle\,, \quad \left\langle\hat{x}\middle|\hat{\phi}\right\rangle, \quad \left\langle\hat{y}\middle|\hat{\phi}\right\rangle$$

(b) Given a vector written in the polar basis

$$|\vec{v}\rangle = a\,|\hat{s}\rangle + b\left|\hat{\phi}\right\rangle$$

where a and b are known.

Express $|\vec{v}\rangle$ in the Cartesian basis,

$$|\vec{v}\rangle = c\,|\hat{x}\rangle + d\,|\hat{y}\rangle$$

by finding c and d

Hint: Use the completeness relation: $|\hat{x}\rangle\langle\hat{x}| + |\hat{y}\rangle\langle\hat{y}| = 1$

(c) Given a quantum state written in the S_z basis,

$$|\Psi\rangle = g\,|+\rangle + h\,|-\rangle\,,$$

express $|\Psi\rangle$ in the S_y basis. That is, find coefficients j and k such that

$$|\Psi\rangle = j\,|+\rangle_y + k\,|-\rangle_y\,.$$