

1 Completeness Relation Change of Basis

(a) Given the polar basis kets written as a superposition of Cartesian kets

$$\begin{aligned} |\hat{s}\rangle &= \cos \phi |\hat{x}\rangle + \sin \phi |\hat{y}\rangle \\ |\hat{\phi}\rangle &= -\sin \phi |\hat{x}\rangle + \cos \phi |\hat{y}\rangle \end{aligned}$$

Find the following quantities:

$$\langle \hat{x} | \hat{s} \rangle, \quad \langle \hat{y} | \hat{s} \rangle, \quad \langle \hat{x} | \hat{\phi} \rangle, \quad \langle \hat{y} | \hat{\phi} \rangle$$

(b) Given a vector written in the polar basis

$$|\vec{v}\rangle = a |\hat{s}\rangle + b |\hat{\phi}\rangle$$

where a and b are known. Find coefficients c and d such that

$$|\vec{v}\rangle = c |\hat{x}\rangle + d |\hat{y}\rangle$$

Do this by using the completeness relation:

$$|\hat{x}\rangle \langle \hat{x}| + |\hat{y}\rangle \langle \hat{y}| = 1$$

(c) Using a completeness relation, change the basis of the spin-1/2 state

$$|\Psi\rangle = g |+\rangle + h |-\rangle$$

into the S_y basis. In otherwords, find j and k such that

$$|\Psi\rangle = j |+\rangle_y + k |-\rangle_y$$