1 Hydrogen Atom Table

Attached, you will find a table showing different representations of physical quantities associated with a the quantum mechanics of the hydrogen atom. Fill in all of the missing entries. Hint: You may look ahead. We filled out a number of the entries throughout the table to give you hints about what the forms of the other entries might be. Be sure to pay close attention to degeneracies, how you index any sums, and how you use label the rows and columns in the matrix representation.

Hydrogen Atom

	Ket Representation	Wave Function Representation	Matrix Representation
Hamiltonian	$\hat{H}$	$\hat{H} = -\frac{\hbar^2}{2\mu}\nabla^2 - \frac{e^2}{4\pi\epsilon_0 r}$ $= -\frac{\hbar^2}{2\mu}\left(\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) - \frac{1}{\hbar^2 r^2}L^2\right) - \frac{e^2}{4\pi\epsilon_0 r}$	
Eigenvalues of Hamiltonian			$E_n = -\frac{\hbar^2}{2\mu a_0^2}\frac{1}{n^2} = -13.6eV\frac{1}{n^2}$
Normalized Eigenstates of Hamiltonian	$\|n,\ell,m\rangle$	$R_n^{\ell}(r)Y_\ell^m(\theta,\phi)$ $= \sqrt{\left(\frac{2}{na_0}\right)^3 \frac{(n-\ell-1)!}{2n((n+\ell)!)^3}}\left(\frac{2r}{na_0}\right)^{\ell} e^{\frac{-r}{na_0}} L_{n+\ell}^{2\ell+1}\left(\frac{2r}{na_0}\right) Y_\ell^m(\theta,\phi)$	
Coefficient of the energy eigenstate with quantum numbers n,ℓ,m	$c_{n,\ell,m} = \langle n,\ell,m\|\Psi\rangle$		$\begin{pmatrix}\cdots & 1 & \cdots\end{pmatrix}\begin{pmatrix}\vdots \\ c_{n,\ell,m} \\ \vdots\end{pmatrix}$
Probability of measuring E_n		$\sum_{\ell=0}^{n-1}\sum_{m=-\ell}^{\ell}\left\|\int_0^{2\pi}\int_0^{\pi}\int_0^{\infty} R_n^{\ell}(r)^* Y_\ell^m(\theta,\phi)^* \Psi(r,\theta,\phi) r^2 \sin\theta\, dr\, d\theta\, d\phi\right\|^2$	$\mathcal{P}(E_n) = \sum_{\ell=0}^{n-1}\sum_{m=-\ell}^{\ell}\left\|\begin{pmatrix}\cdots & 1 & \cdots\end{pmatrix}\begin{pmatrix}\vdots \\ c_{n,\ell,m} \\ \vdots\end{pmatrix}\right\|^2$

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	Ket Representation	Wave Function Representation	Matrix Representation
Operator for square of the angular momentum	L^2	$L^2 = -\hbar^2\left(\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right)+\frac{1}{\sin^2\theta}\frac{d^2}{d\phi^2}\right)$ $= -\hbar^2\left(\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right)+\frac{1}{\hbar^2\sin^2\theta}L_z^{\,2}\right)$	
Eigenvalues of L^2			$\hbar^2\ell(\ell+1)$
Normalized Eigenstates of L^2	$\lvert n,\ell,m\rangle$		$\lvert 1,0,0\rangle \doteq \begin{pmatrix}1\\0\\0\\0\\\vdots\end{pmatrix}, \lvert 2,0,0\rangle \doteq \begin{pmatrix}0\\1\\0\\0\\\vdots\end{pmatrix}, \lvert 2,1,1\rangle \doteq \begin{pmatrix}0\\0\\1\\0\\\vdots\end{pmatrix}, \lvert 2,1,0\rangle \doteq \begin{pmatrix}0\\0\\0\\1\\\vdots\end{pmatrix},$
Coefficient of the eigenstates of L^2 with quantum numbers n,ℓ,m		$c_{n,\ell,m} = \int_0^{2\pi}\int_0^{\pi}\int_0^{\infty} R_n^{\ell}(r)^* Y_\ell^m(\theta,\phi)^*\,\Psi(r,\theta,\phi)\,r^2\sin\theta\,dr\,d\theta\,d\phi$	$\begin{pmatrix}\cdots & 1 & \cdots\end{pmatrix}\begin{pmatrix}\vdots\\c_{n,\ell,m}\\\vdots\end{pmatrix}$
Probability of measuring $\hbar^2\ell(\ell+1)$ for the square of the angular momentum	$\mathcal{P}\left(\hbar^2\ell(\ell+1)\right)$ $= \sum_{n=\ell+1}^{\infty}\sum_{m=-\ell}^{\ell}\lvert c_{n,\ell,m}\rvert^2$ $= \sum_{n=\ell+1}^{\infty}\sum_{m=-\ell}^{\ell}\lvert\langle n,\ell,m\vert\Psi\rangle\rvert^2$		$\mathcal{P}\left(\hbar^2\ell(\ell+1)\right) = \sum_{n=\ell+1}^{\infty}\sum_{m=-\ell}^{\ell}\left\lvert\begin{pmatrix}\cdots & 1 & \cdots\end{pmatrix}\begin{pmatrix}\vdots\\c_{n,\ell,m}\\\vdots\end{pmatrix}\right\rvert^2$

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	Ket Representation	Wave Function Representation	Matrix Representation
Operator for z-component of angular momentum	L_z		$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & \cdots \\ 0 & 0 & 0 & 0 & 0 & 0 & \cdots \\ 0 & 0 & \hbar & 0 & 0 & 0 & \cdots \\ 0 & 0 & 0 & 0 & 0 & 0 & \cdots \\ 0 & 0 & 0 & 0 & -\hbar & 0 & \cdots \\ 0 & 0 & 0 & 0 & 0 & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$
Eigenstates of L_z		$m\hbar$	
Normalized Eigenstates of L_z	|	$R_n^{\ell}(r)Y_\ell^m(\theta,\phi) = \sqrt{\left(\frac{2}{na_0}\right)^3 \frac{(n-\ell-1)!}{2n\left((n+\ell)!\right)^3}} \left(\frac{2r}{na_0}\right)^{\ell} e^{\frac{-r}{na_0}} L_{n+\ell}^{2\ell+1}\left(\frac{2r}{na_0}\right) Y_\ell^m(\theta,\phi)$	$\|1,0,0\rangle \doteq \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ \vdots \end{pmatrix}, \|2,0,0\rangle \doteq \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ \vdots \end{pmatrix}, \|2,1,1\rangle \doteq \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ \vdots \end{pmatrix}, \|2,1,0\rangle \doteq \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ \vdots \end{pmatrix}, \ldots$
Coefficient of m^{th} eigenstates of L_z	$c_{n,\ell,m} = \langle n,\ell,m\|\Psi\rangle$	$c_{n,\ell,m} = \int_0^{2\pi}\int_0^{\pi}\int_0^{\infty} R_n^{\ell}(r)^* Y_\ell^m(\theta,\phi)^* \Psi(r,\theta,\phi) r^2 \sin\theta dr d\theta d\phi$	
Probability of measuring $m\hbar$ for z-component of angular momentum	$\mathcal{P}(m\hbar) = \sum_{n=\|m\|+1}^{\infty} \sum_{\ell=\|m\|}^{\infty} \left\|c_{n,\ell,m}\right\|^2 = \sum_{n=\|m\|+1}^{\infty} \sum_{\ell=\|m\|}^{\infty} \left\|\langle n,\ell,m\|\Psi\rangle\right\|^2$	$\sum_{n=\|m\|+1}^{\infty} \sum_{\ell=\|m\|}^{\infty} \left\| \int_0^{2\pi}\int_0^{\pi}\int_0^{\infty} R_n^{\ell}(r)^* Y_\ell^m(\theta,\phi)^* \Psi(r,\theta,\phi) r^2 \sin\theta dr d\theta d\phi \right\|^2$	