1 Energy, Entropy, and Probabilities

The goal of this problem is to show that once we have maximized the entropy and found the microstate probabilities in terms of a Lagrange multiplier β, we can prove that $\beta = \frac{1}{kT}$ based on the statistical definitions of energy and entropy and the thermodynamic definition of temperature embodied in the thermodynamic identity.

The internal energy and entropy are each defined as a weighted average over microstates:

$$U = \sum_i E_i P_i \qquad S = -k_B \sum_i P_i \ln P_i \tag{1}$$

: We saw in clase that the probability of each microstate can be given in terms of a Lagrange multiplier β as

$$P_i = \frac{e^{-\beta E_i}}{Z} \qquad Z = \sum_i e^{-\beta E_i} \tag{2}$$

Put these probabilities into the above weighted averages in order to relate U and S to β. Then make use of the thermodynamic identity

$$dU = TdS - pdV \tag{3}$$

to show that $\beta = \frac{1}{kT}$.