

1 Gibbs sum for a two level system

- (a) Consider a system that may be unoccupied with energy zero, or occupied by one particle in either of two states, one of energy zero and one of energy ε . Find the Gibbs sum for this system is in terms of the activity $\lambda \equiv e^{\beta\mu}$. Note that the system can hold a maximum of one particle.
- (b) Solve for the thermal average occupancy of the system in terms of λ .
- (c) Show that the thermal average occupancy of the state at energy ε is

$$\langle N(\varepsilon) \rangle = \frac{\lambda e^{-\frac{\varepsilon}{kT}}}{\mathcal{Z}} \quad (1)$$

- (d) Find an expression for the thermal average energy of the system.
- (e) Allow the possibility that the orbitals at 0 and at ε may each be occupied each by one particle at the same time; Show that

$$\mathcal{Z} = 1 + \lambda + \lambda e^{-\frac{\varepsilon}{kT}} + \lambda^2 e^{-\frac{\varepsilon}{kT}} \quad (2)$$

$$= (1 + \lambda) (1 + e^{-\frac{\varepsilon}{kT}}) \quad (3)$$

Because \mathcal{Z} can be factored as shown, we have in effect two independent systems.