

# 1 Einstein condensation temperature

**Einstein condensation temperature** Starting from the density of free particle orbitals per unit energy range

$$\mathcal{D}(\varepsilon) = \frac{V}{4\pi^2} \left( \frac{2M}{\hbar^2} \right)^{\frac{3}{2}} \varepsilon^{\frac{1}{2}} \quad (1)$$

show that the lowest temperature at which the total number of atoms in excited states is equal to the total number of atoms is

$$T_E = \frac{1}{k_B} \frac{\hbar^2}{2M} \left( \frac{N}{V} \frac{4\pi^2}{\int_0^\infty \frac{\sqrt{\xi}}{e^\xi - 1} d\xi} \right)^{\frac{2}{3}} T_E = \quad (2)$$

The infinite sum may be numerically evaluated to be 2.612. Note that the number derived by integrating over the density of states, since the density of states includes all the states *except* the ground state.

**Note:** This problem is solved in the text itself. I intend to discuss Bose-Einstein condensation in class, but will not derive this result.