Time Evolution of a Spin-1/2 System

Consider a spin-1/2 system with a Hamiltonain that is proportional to $\hat{S}_z$:

$$\hat{H} = \omega_0 \hat{S}_z$$

At $t = 0$:

1. one particle is in the state $|+\rangle_x$.
2. another particle is in the state $|+\rangle$

For each particle:

1. What values of energy could you measure?
2. What are the energy eigenstates?
3. What state is each particle in at a later time t?
4. What are the probabilities for each energy measurement?
5. What is the probability that you would measure $S_x = \frac{\hbar}{2}$ state at time t? Does this probability change with time?
6. What is the probability that you would measure $S_z = \frac{\hbar}{2}$ at time t? Does this probability change with time?
7. Given a Hamiltonian, how would you determine which states are stationary states (states where no probabilities change with time)? Under what circumstances do measurement probabilities change with time?