Eigenvalues and Eigenvectors

Each group will be assigned one of the following matrices.

$$A_1 \doteq \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \qquad A_2 \doteq \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad A_3 \doteq \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$A_4 \doteq \begin{pmatrix} a & 0 \\ 0 & d \end{pmatrix} \qquad A_5 \doteq \begin{pmatrix} 3 & -i \\ i & 3 \end{pmatrix} \qquad A_6 \doteq \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \qquad A_7 \doteq \begin{pmatrix} 1 & 2 \\ 1 & 2 \end{pmatrix}$$

$$A_8 \doteq \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \qquad A_9 \doteq \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$S_x \doteq \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad S_y \doteq \frac{\hbar}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad S_z \doteq \frac{\hbar}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

For your matrix:

1. Find the eigenvalues.
2. Find the (unnormalized) eigenvectors.
3. Describe what this transformation does.
4. Normalize your eigenstates.

If you finish early, try another matrix with a different structure, *i.e.* real vs. complex entries, diagonal vs. non-diagonal, 2×2 vs. 3×3, with vs. without explicit dimensions.