Linear Transformations

1. Using colored markers, draw these initial vectors, all on the same graph on your whiteboard.

$$\vec{v}_{red} = 1|\hat{x}\rangle + 0|\hat{y}\rangle \doteq \begin{pmatrix}1\\0\end{pmatrix} \qquad \vec{v}_{green} = 0|\hat{x}\rangle + 1|\hat{y}\rangle \doteq \begin{pmatrix}0\\1\end{pmatrix} \qquad \vec{v}_{blue} = 1|\hat{x}\rangle + 1|\hat{y}\rangle \doteq \begin{pmatrix}1\\1\end{pmatrix}$$

$$\vec{v}_{black} = 1|\hat{x}\rangle - 1|\hat{y}\rangle \doteq \begin{pmatrix}1\\-1\end{pmatrix} \qquad \vec{v}_{purple} = 1|\hat{x}\rangle + 3|\hat{y}\rangle \doteq \begin{pmatrix}1\\3\end{pmatrix}$$

2. Each group will be assigned one of the following matrices. Operate on the initial vectors with your group's matrix and graph the transformed vectors on a single (new) graph.

$$A_1 \doteq \begin{pmatrix}0 & 1\\-1 & 0\end{pmatrix} \quad A_2 \doteq \begin{pmatrix}0 & -1\\1 & 0\end{pmatrix} \quad A_3 \doteq \begin{pmatrix}0 & 1\\1 & 0\end{pmatrix} \quad A_4 \doteq \begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}$$

$$A_5 \doteq \begin{pmatrix}-1 & 0\\0 & -1\end{pmatrix} \quad A_6 \doteq \begin{pmatrix}1 & 2\\1 & 2\end{pmatrix} \quad A_7 \doteq \begin{pmatrix}1 & 2\\9 & 4\end{pmatrix} \quad A_8 \doteq \begin{pmatrix}1 & 1\\-1 & 1\end{pmatrix}$$

$$A_9 \doteq \begin{pmatrix}2 & 0\\0 & 2\end{pmatrix} \quad A_{10} \doteq \begin{pmatrix}1 & 1\\1 & 1\end{pmatrix} \quad A_{11} \doteq \begin{pmatrix}1 & 0\\0 & 0\end{pmatrix} \quad A_{12} \doteq \frac{\hbar}{2}\begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}$$

3. Find the determinant of your matrix.

4. Make note of any differences between the initial and transformed vectors. Specifically, look for rotations, inversions, length changes, anything that is different. Are there any vectors which are left unchanged by your transformation? Your group should be prepared to report to the class about your transformation.

5. After all groups are done, you will be reporting to the class:

   - What is your matrix (give the number and read the elements from top left to bottom right)
   - What is the determinant of your matrix?
   - What does your matrix do (geometrically) to vectors?
   - Are there any vectors that don't change direction?

6. If you get done early, save your original board for the report and work on some other examples that are structurally different from your original example.