

Representations of the Infinite Square Well

Consider three particles of mass m which are each in an infinite square well potential at $0 < x < L$. The energy eigenstates of the infinite square well are:

$$\langle x|E_n\rangle = \phi_n(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right)$$

with energy eigenvalues $E_n = \frac{n^2\pi^2\hbar^2}{2mL^2}$

The particles are initially in the states, respectively:

$$\begin{aligned} |\psi_a(0)\rangle &= A \left[|E_1\rangle + 2i|E_4\rangle - 3|E_{10}\rangle \right] \\ \psi_b(x, 0) &= B \left[\sqrt{\frac{2}{L}} \sin\left(\frac{\pi x}{L}\right) + i\sqrt{\frac{8}{L}} \sin\left(\frac{4\pi x}{L}\right) - \sqrt{\frac{18}{L}} \sin\left(\frac{10\pi x}{L}\right) \right] \\ \psi_c(x, 0) &= Cx(x - L) \end{aligned}$$

For each particle:

1. Determine the value of the normalization constant.
2. At $t = 0$, what is the probability of measuring the energy of the particle to be $\frac{8\pi^2\hbar^2}{mL^2}$?
3. Find the state of the particle at a later time t .
4. What is the probability of measuring the energy of the particle to be the same value $\frac{8\pi^2\hbar^2}{mL^2}$ at a later time t ?
5. What is the probability of finding the particle to be in the left half of the well?