Commutation Relations for Spin Operators

A commutator of two observables is defined as:

$$[\hat{A}, \hat{B}] = \hat{A}\hat{B} - \hat{B}\hat{A}$$

Determine the results of the following commutators:

1. $[\hat{S}_x, \hat{S}_y]$

2. $[\hat{S}_y, \hat{S}_z]$

3. $[\hat{S}_z, \hat{S}_x]$

4. $[\hat{S}_y, \hat{S}_x]$

5. $[\hat{S}_z, \hat{S}_y]$

6. $[\hat{S}_x, \hat{S}_z]$

Remember that the matrix representation of the spin operators written in the S_z basis is:

$$\hat{S}_x \doteq \frac{\hbar}{2}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad \hat{S}_y \doteq \frac{\hbar}{2}\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \quad \hat{S}_z \doteq \frac{\hbar}{2}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$