Outer Product of a Vector on Itself

1. Your group will be given a pair (or triple) of vectors below, find the matrix that is the outer product of each vector on itself (i.e., $|v_1\rangle\langle v_1|$)? All the vectors are written in the S_z basis.

1) $|+\rangle \doteq \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $\quad |-\rangle \doteq \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

2) $|+\rangle_x \doteq \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ $\quad |-\rangle_x \doteq \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

3) $|+\rangle_y \doteq \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ i \end{bmatrix}$ $\quad |-\rangle_y \doteq \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -i \end{bmatrix}$

4) $|v_7\rangle \doteq \frac{1}{5} \begin{bmatrix} 3 \\ 4 \end{bmatrix}$ $\quad |v_8\rangle \doteq \frac{1}{5} \begin{bmatrix} 4 \\ -3 \end{bmatrix}$

5) $|v_9\rangle \doteq \begin{bmatrix} a \\ be^{i\phi} \end{bmatrix}$ $\quad |v_{10}\rangle \doteq \begin{bmatrix} b \\ -ae^{i\phi} \end{bmatrix}$

6) $|1\rangle_x \doteq \frac{1}{\sqrt{2}} \begin{bmatrix} \frac{1}{\sqrt{2}} \\ 1 \\ \frac{1}{\sqrt{2}} \end{bmatrix}$ $\quad |0\rangle_x \doteq \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$ $\quad |-1\rangle_x \doteq \frac{1}{\sqrt{2}} \begin{bmatrix} \frac{1}{\sqrt{2}} \\ -1 \\ \frac{1}{\sqrt{2}} \end{bmatrix}$

2. What is the square of each of your outer products?
3. What is the product of each pair of your outer products?
4. For each row of vectors, add all of the outer products.
5. What is the determinant of each of your outer products?
6. What is the transformation caused by each of your outer products?

   Bonus: How would you answer questions (2), (3), (4) staying purely in Dirac bra-ket notation?