Consider the vector field given by *(μ_0 and I are constants)*: $\vec{\boldsymbol{B}} = \frac{\mu_0 I}{2\pi}\left(\frac{-y\,\hat{\boldsymbol{x}}+x\,\hat{\boldsymbol{y}}}{x^2+y^2}\right) = \frac{\mu_0 I}{2\pi}\,\frac{\hat{\boldsymbol{\phi}}}{s}$

$\vec{\boldsymbol{B}}$ *is the magnetic field around a wire along the z-axis carrying a constant current I in the z-direction.*

Ready:

- Determine $\vec{\boldsymbol{B}} \cdot d\vec{\boldsymbol{r}}$ on any radial line of the form $y = mx$, where m is a constant.
- Determine $\vec{\boldsymbol{B}} \cdot d\vec{\boldsymbol{r}}$ on any circle of the form $x^2 + y^2 = a^2$, where a is a constant.
  You may wish to express the equations for these curves in polar coordinates.

Go: For each of the following curves C_i, evaluate the line integral $\int_{C_i} \vec{\boldsymbol{B}} \cdot d\vec{\boldsymbol{r}}$.

- C_1, the *top* half of the circle $s = 5$, traversed in a *counterclockwise* direction.
- C_2, the *top* half of the circle $s = 2$, traversed in a *counterclockwise* direction.
- C_3, the *top* half of the circle $s = 2$, traversed in a *clockwise* direction.
- C_4, the *bottom* half of the circle $s = 2$, traversed in a *clockwise* direction.
- C_5, the radial line from $(2, 0)$ to $(5, 0)$.
- C_6, the radial line from $(-5, 0)$ to $(-2, 0)$.

FOOD FOR THOUGHT

- Construct ***closed*** curves C_7 and C_8 such that this integral $\int_{C_i} \vec{\boldsymbol{B}} \cdot d\vec{\boldsymbol{r}}$ is nonzero over C_7 and zero over C_8.
  *It is enough to draw your curves; you do **not** need to parameterize them.*
- Ampère's Law says that, for any closed curve C, this integral is (μ_0 times) the current flowing ***through*** C (in the z direction). Can you use this fact to explain your results to part (a)?
- Is $\vec{\boldsymbol{B}}$ conservative?