Topics are everything that has been covered on homework. Problems should be similar to homework problems, but short enough to be completed during the exam. The exam will be closed notes. You should be able to remember the fundamental equations.

0.1 Equations to remember

Most of the equations I expect you to remember date back from Energy and Entropy, with a few exceptions.

Thermodynamic identity The thermodynamic identity, including the chemical potential:

$$dU = TdS - pdV + \mu dN \tag{1}$$

You should be able from this to extract relationships such as $\mu = \left(\frac{\partial U}{\partial N}\right)_{S,V}$.

Thermodynamic potentials You need to know the Helmholtz and Gibbs free energies.

$$F = U - TS \tag{2}$$
$$G = U - TS + pV \tag{3}$$
$$dF = -SdT - pdV + \mu dN \tag{4}$$
$$dG = -SdT + Vdp + \mu dN \tag{5}$$

You don't need to remember their differentials, but you do need to be able to find them quickly and use them, e.g. to find out how μ relates to F as a derivative. I'll point out that by remembering how to find the differentials, you also don't need to remember the sign of $U - TS$, since you can figure it out from the thermodynamic identity by making the TdS term cancel.

Heat and work You should remember the expressions for differential heat and work

$$dQ = TdS \tag{6}$$
$$dW = -pdV \tag{7}$$

and you should be able to use these expressions fluently, including integrating to find total heat or work, or solving for entropy given heat:

$$dS = \frac{dQ}{T} \tag{8}$$

Efficiency You should know that efficiency is defined as "what you get out" divided by "what you put in", and that for a heat engine this comes down to

$$\epsilon = \frac{W_{\text{net}}}{Q_H} \tag{9}$$

Entropy You should remember the Gibbs expression for entropy in terms of probability.

$$S = -k \sum_i P_i \ln P_i \tag{10}$$

Boltzmann probability You should be comfortable with the Boltzmann probability, able to predict properties of systems using them.

$$P_i = \frac{e^{-\beta E_i}}{Z} \tag{11}$$
$$Z = \sum_i e^{-\beta E_i} \tag{12}$$
$$F = -kT \ln Z \tag{13}$$

Derivative trick You *may need* to remember the derivative trick for turning a summation into a derivative of another summation in order to complete a problem. More particularly, I want you **not to use** an expression for U in terms of Z that comes from the derivative trick, without writing down the three lines of math (or so) required to show that it is true.

Thermal averages You should remember that the internal energy is given by a weighted average:

$$U = \sum_i E_i P_i \tag{14}$$

And similarly for other variables, such as N in the grand canonical ensemble.

Chemical potential You should remember that the chemical potential is the Gibbs free energy per particle.

$$\mu = \frac{G}{N} \tag{15}$$

You should also be able to make a distinction between internal and external chemical potential to solve problems such as finding the density as a function of altitude (or in a centrifuge), if I give you the expression for the chemical potential of an ideal gas (or other fluid).

Gibbs factor and sum You should be comfortable with the Gibbs sum and finding probabilities in the grand canonical ensemble.

$$P_i = \frac{e^{-\beta(E_i - \mu N_i)}}{Z} \tag{16}$$
$$\mathcal{Z} = \sum_i e^{-\beta(E_i - \mu N_i)} \tag{17}$$

Incidentally, in class we didn't cover the grand potential (or grand free energy), but that is what you get if you try to find a free energy using the Gibbs sum like the partition function.

Fermi-Dirac, Bose-Einstein, and Planck distributions You should remember these distributions

$$f_{FD}(\varepsilon) = \frac{1}{e^{\beta(\varepsilon-\mu)} + 1} \tag{18}$$
$$f_{BE}(\varepsilon) = \frac{1}{e^{\beta(\varepsilon-\mu)} - 1} \tag{19}$$

and should be able to use them to make predictions for properties of non-interacting systems of fermions and bosons. This also requires remembering how to reason about orbitals as essentially independent systems within the grand canonical ensemble. You should remember that the Planck distribution for photons (or phonons) is the same as the Bose-Einstein distribution, but with $\mu = 0$. This comes about because photons and phonons are bosons, but are a special kind of boson that has no conservation of particle number.

Density of states You should remember how to use a density of states together with the above distributions to find properties of a system of noninteracting fermions or bosons

$$\langle X(\varepsilon)\rangle = \int \mathcal{D}(\varepsilon) f(\varepsilon) X(\varepsilon) d\varepsilon \tag{20}$$

As special cases of this, you should be able to find N (or given N find μ) or the internal energy. We had a few homeworks where you found entropy from the density of states, but I think that was a bit too challenging/confusing to put on the final exam.

Conditions for coexistence You should remember that when two phases are in coexistence, their temperatures, pressures, and chemical potentials must be identical, and you should be able to make use of this.

0.2 Equations not to remember

If you need a property of a particular system (the ideal gas, the simple harmonic oscillator), it will be given to you. There is no need, for instance, to remember the Stefan-Boltzmann law or the Planck distribution.

Heat capacity I do not expect you to remember the definition of heat capacity (although you probably will remember it).

$$C_V = T\left(\frac{\partial S}{\partial T}\right)_{V,N} \tag{21}$$

$$= \left(\frac{\partial U}{\partial T}\right)_{V,N} \tag{22}$$

$$C_p = T\left(\frac{\partial S}{\partial T}\right)_{p,N} \tag{23}$$

I do expect you to be able to make use of these equations when given. Similarly, you should be able to show that the two expressions for C_V are equal, using the thermodynamic identity.

Enthalpy If I give you the expression for enthalpy $(U + pV)$ you should be able to work with it, but since we didn't touch it in class, I don't expect you to remember what it is.

Any property of an ideal gas I don't expect you to remember any property of an ideal gas, including its pressure (i.e. ideal gas law), free energy, entropy, internal energy, or chemical potential. You should be comfortable with these expressions, however, and if I provide them should be able to make use of them.

Stefan-Boltzmann equation You should be able to make use of the expression that

$$I = \sigma_B T^4 \tag{24}$$

where I is the power radiated per area of surface, but need not remember this.

Clausius-Clapeyron equation You should be able to make use of

$$\frac{dp}{dT} = \frac{s_g - s_\ell}{v_g - v_\ell} \tag{25}$$

but I don't expect you to remember this. You should also be able to convert between this expression and the one involving latent heat using your knowledge of heat and entropy.

Carnot efficiency You need not remember the Carnot efficiency

$$\epsilon = 1 - \frac{T_C}{T_H} \tag{26}$$

but you should remember what an efficiency is, and should be able to pretty quickly solve for the net work and high-temperature heat for a Carnot engine by looking at it in T/S space. (Or similarly for a Carnot refridgerator.)

Density of states for particular systems You need not remember any expression for the density of states e.g. for a gas. But given such an expression, you should be able to make use of it.

Fermi energy You need not remember any particular expression for the Fermi energy of a particular system, but should be able to make use of an expression for the Fermi energy of a system.