Consider the following normalized quantum state on a unit ring:

$$\Phi(\phi) = \sqrt{\frac{8}{3\pi r_0}} \sin^2(3\,\phi)\cos(\phi) \tag{1}$$

1. Express this state in the L_z basis, using three representations: wavefunction, bra/ket, and matrix representations. Remember that you can use any of these representations in the following calculations.

2. What is the expectation value of L_z in this state?

3. What is the probability that the particle can be found in the region $0 < \phi < \frac{\pi}{4}$? Repeat your calculation in the region $\frac{\pi}{4} < \phi < \frac{3\pi}{4}$?