

For  $\ell = 1$ , the operators that measure the three components of angular momentum in matrix notation are given by:

$$L_x = \frac{\hbar}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad (1)$$

$$L_y = \frac{\hbar}{\sqrt{2}} \begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix} \quad (2)$$

$$L_z = \hbar \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix} \quad (3)$$

Show that:

1. Find the commutator of  $L_x$  and  $L_y$ .
2. Find the matrix representation of  $L^2 = L_x^2 + L_y^2 + L_z^2$ .
3. Find the matrix representations of the raising and lowering operators  $L_{\pm} = L_x \pm iL_y$ . (Notice that  $L_{\pm}$  are NOT Hermitian and therefore cannot represent observables. They are used as a tool to build one quantum state from another.)
4. Show that  $[L_z, L_{\pm}] = \lambda L_{\pm}$ . Find  $\lambda$ . Interpret this expression as an eigenvalue equation. What is the operator?
5. Let  $L_+$  act on the following three states given in matrix representation.

$$|1, 1\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad |1, 0\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad |1, -1\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \quad (4)$$

Why is  $L_+$  called a “raising operator”?