For $\ell = 1$, the operators that measure the three components of angular momentum in matrix notation are given by:

$$L_x = \frac{\hbar}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \tag{1}$$

$$L_y = \frac{\hbar}{\sqrt{2}} \begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix} \tag{2}$$

$$L_z = \hbar \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix} \tag{3}$$

Show that:

1. Find the commutator of L_x and L_y.
2. Find the matrix representation of $L^2 = L_x^2 + L_y^2 + L_z^2$.
3. Find the matrix representations of the raising and lowering operators $L_\pm = L_x \pm iL_y$. (Notice that $L_\pm$ are NOT Hermitian and therefore cannot represent observables. They are used as a tool to build one quantum state from another.)
4. Show that $[L_z, L_\pm] = \lambda L_\pm$. Find λ. Interpret this expression as an eigenvalue equation. What is the operator?
5. Let L_+ act on the following three states given in matrix representation.

$$|1,1\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \qquad |1,0\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \qquad |1,-1\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \tag{4}$$

Why is L_+ called a "raising operator"?