1 Instructor's Guide

1.1 Prerequisite Knowledge

We usually do this activity after giving the students a brief introduction to cylindrical and spherical coordinates (e.g. Curvilinear Coordinates Introduction).

1.2 Student's Task

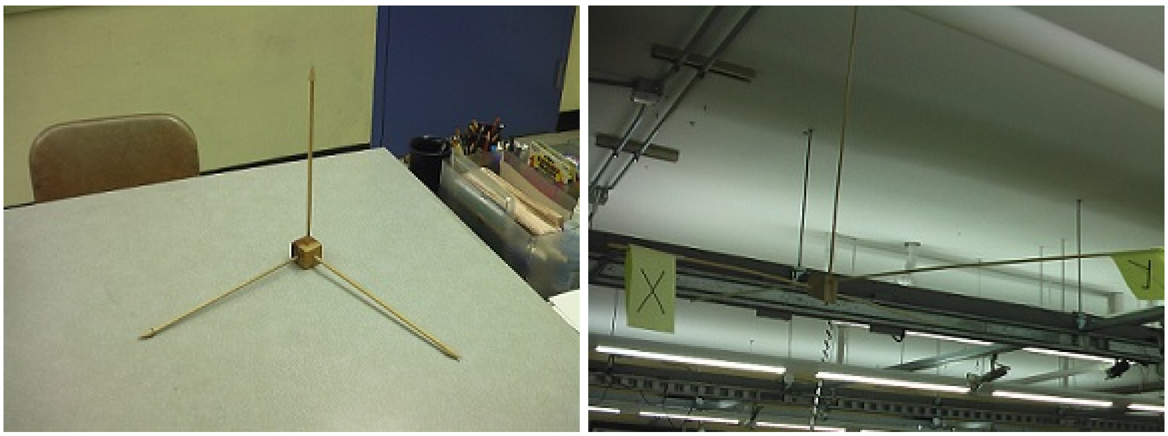
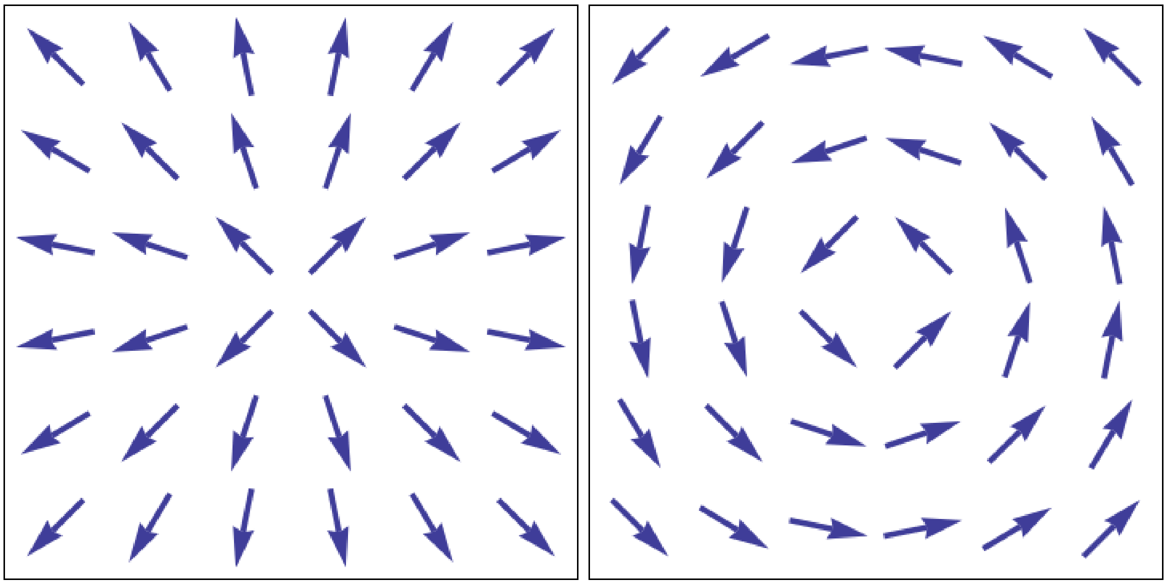


Figure 1: Figure: Coordinate axes, both freestanding and hanging.

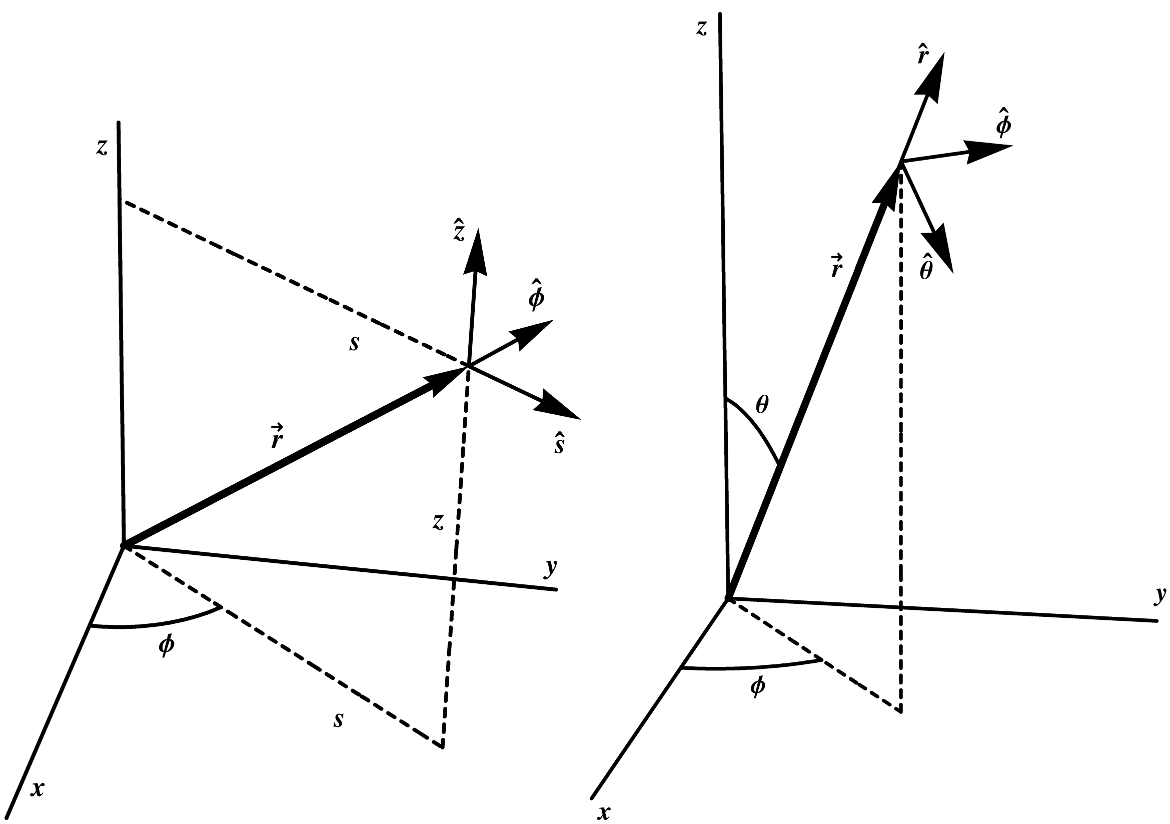
- Place a set of coordinate axes somewhere in the room. We prefer an origin of coordinates on the ceiling in a back corner of the room. If you don't have a set of coordinate axes, then explain that a rectangular piece of furniture such as the podium in the classroom or a corner of the room will represent the origin and rectangular axes.
- Tell the students: "Imagine that your right shoulder is a point in space relative to the origin of coordinates. Use your right arm to show $\hat{x}$." Then, quickly repeat with $\hat{y}$ and $\hat{z}$.
- Now tell the students: "CLOSE YOUR EYES. Use your right arm to show $\hat{s}$ in cylindrical coordinates."
- Once everybody has committed to an answer, ask the students to open their eyes.
- Repeat for $\hat{\phi}$ in cylindrical and $\hat{\theta}$ in spherical coordinates.

1.3 Student Conversations

- **Which direction should the basis vector point?:** A basis vector $\widehat{coordinate}$ is the unit vector that points in the direction that *coordinate* is changing, i.e. $\hat{x}$ is the unit vector that points in the direction that x is changing.
- **Basis vectors are not generally constant in space:** Although the basis vectors that correspond to rectangular coordinates x, y, and z are constants, i.e. they point the same direction at each point in space; most of the basis vectors that correspond to cylindrical and spherical coordinates point in different directions at each point in space. When using basis vectors adapted to curvilinear coordinates in derivatives and integrals, it is essential to remember that these basis vectors are not constant.
- **Curvilinear basis vectors make a nice example of a vector field:** The basis vectors adapted to a single coordinate form a simple example of the geometrical notion of a vector field, i.e. a vector at every point in space. For example, the polar basis vectors $\hat{r}, \hat{\phi}$ are shown in these figures



- **Radial basis vectors:** In cylindrical coordinates, the radial basis vector $\hat{s}$ is always parallel to the x-y plane (i.e. the floor of your classroom), but in spherical coordinates, the radial basis vector $\hat{r}$ points directly away from the origin (i.e. is only parallel to the floor when evaluated for points in the x-y plane).
- **Basis vectors are straight:** The basis vectors are vectors, i.e. they are *straight* arrows in space, even when they correspond to coordinates that are angles.
- **$\hat{\theta}$ should point generally downward:** Make sure that the directions in which students point agree with the directions in the figures below.



In particular, $\hat{\theta}$ should point generally downward.

- **The basis vectors at a single point form an orthonormal set:** The basis vectors for cylindrical or spherical coordinates at a single point are orthonormal. Therefore, they can be used to expand any vector at that point and the formulas for dot and cross product work in exactly the same way that they would for rectangular basis vectors. However, since the directions of the basis vectors vary from point to point, they should NEVER be used to compare two vectors that live at different points.

1.4 Wrap-up

No wrap-up in needed beyond covering all the points listed in Student Conversations. The student handout solution can be provided to students. It contains figures that show the coordinate basis vectors at a single point.

Student handout You will probably be doing this activity in-class, from directions given by the instructor. If you are doing it on your own, then choose a point in the room that you are in to be the origin. Imagine that your right shoulder is a point in space, relative to that origin. Point your right arm in succession in each of the directions of the basis vectors adapted the various coordinate systems:

- $\{\hat{x}, \hat{y}, \hat{z}\}$ in rectangular coordinates.
- $\left\{\hat{s}, \hat{\phi}, \hat{z}\right\}$ in cylindrical coordinates.
- $\left\{\hat{r}, \hat{\theta}, \hat{\phi}\right\}$ in spherical coordinates.