Student handout

Representations of the Infinite Square Well

Consider three particles of mass m which are each in an infinite square well potential at $0 < x < L$.
The energy eigenstates of the infinite square well are:

$$E_n(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right)$$

with energies $E_n = \frac{n^2\pi^2\hbar^2}{2mL^2}$
The particles are initially in the states, respectively:

$$\begin{aligned}
\left|\psi_a(0)\right\rangle &= A\Big[2i\left|E_4\right\rangle - 3\left|E_{10}\right\rangle\Big] \\
\psi_b(x,0) &= B\left[i\sqrt{\frac{8}{L}}\sin\left(\frac{4\pi x}{L}\right) - \sqrt{\frac{18}{L}}\sin\left(\frac{10\pi x}{L}\right)\right] \\
\psi_c(x,0) &= Cx(x-L)
\end{aligned}$$

For each particle:

1. Determine the normalization constant.
2. At $t = 0$ what is the probability of measuring the energy of the particle to be $\frac{8\pi^2\hbar^2}{mL^2}$?
3. Find state of the particle at a later time t.
4. What is the probability of measuring the energy of the particle to be the same value $\frac{8\pi^2\hbar^2}{mL^2}$ at a later time t?
5. What is the probability of finding the particle to be in the first half of the well?

1 Student Conversations

1. Help students recognize that particle a and particle b are in the same state.
2. For normalization, emphasize that you must calculate the square of the norm of the state BEFORE you integrate.
3. The energy value given is simplified - students need to recognize that this energy corresponds to $n = 4$.
4. Time evolving particle c is brutal for the students. Reassure students that they have to leave it as a sum. Setting up the integral is the point here. For time expediancy, encourage students to leave the integral to be evaluated later.
5. For Hamiltonian's that don't evolve with time, the probabilities of measuring energies are time independent.
6. Emphasize to students that you can't calculate the probability of finding a particle in a region in Dirac notation.