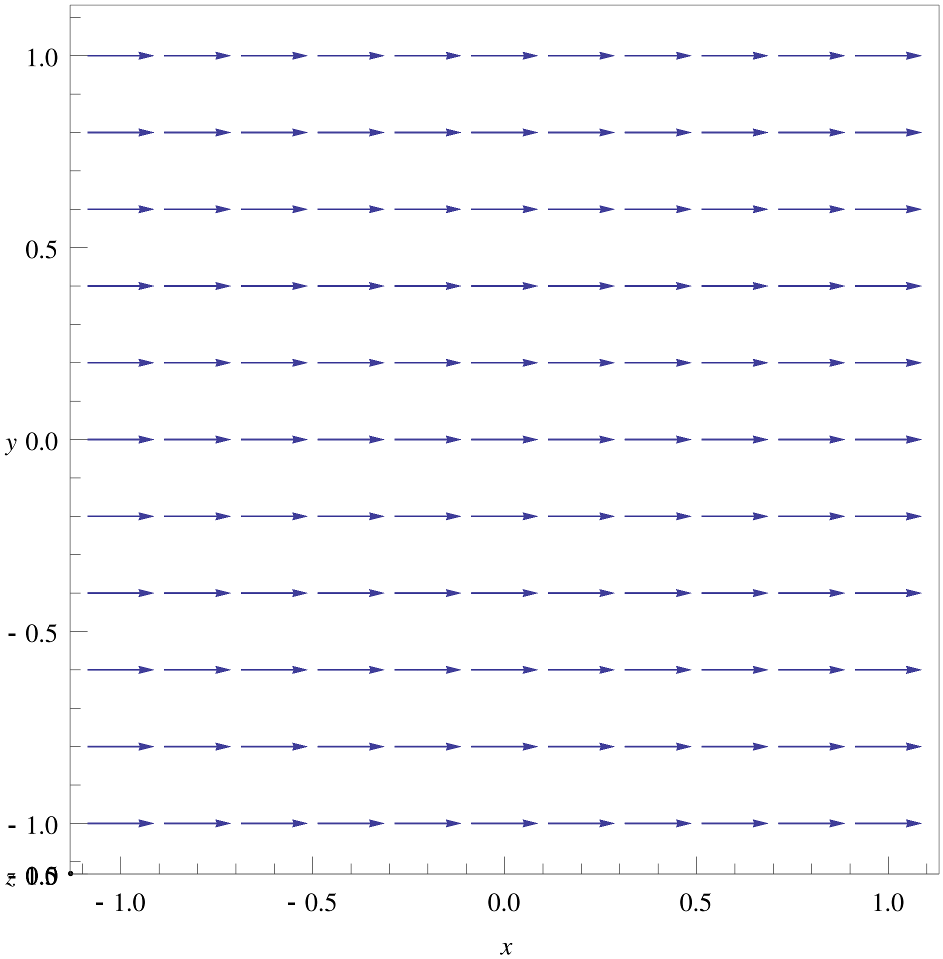
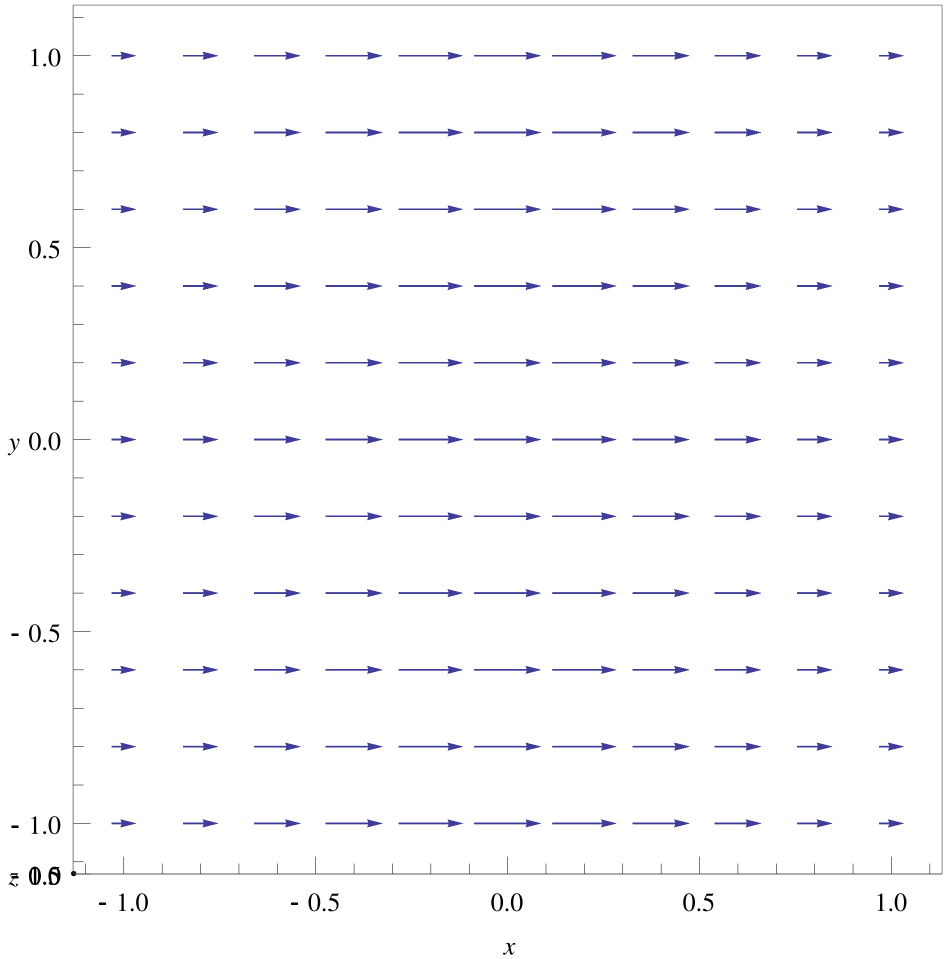
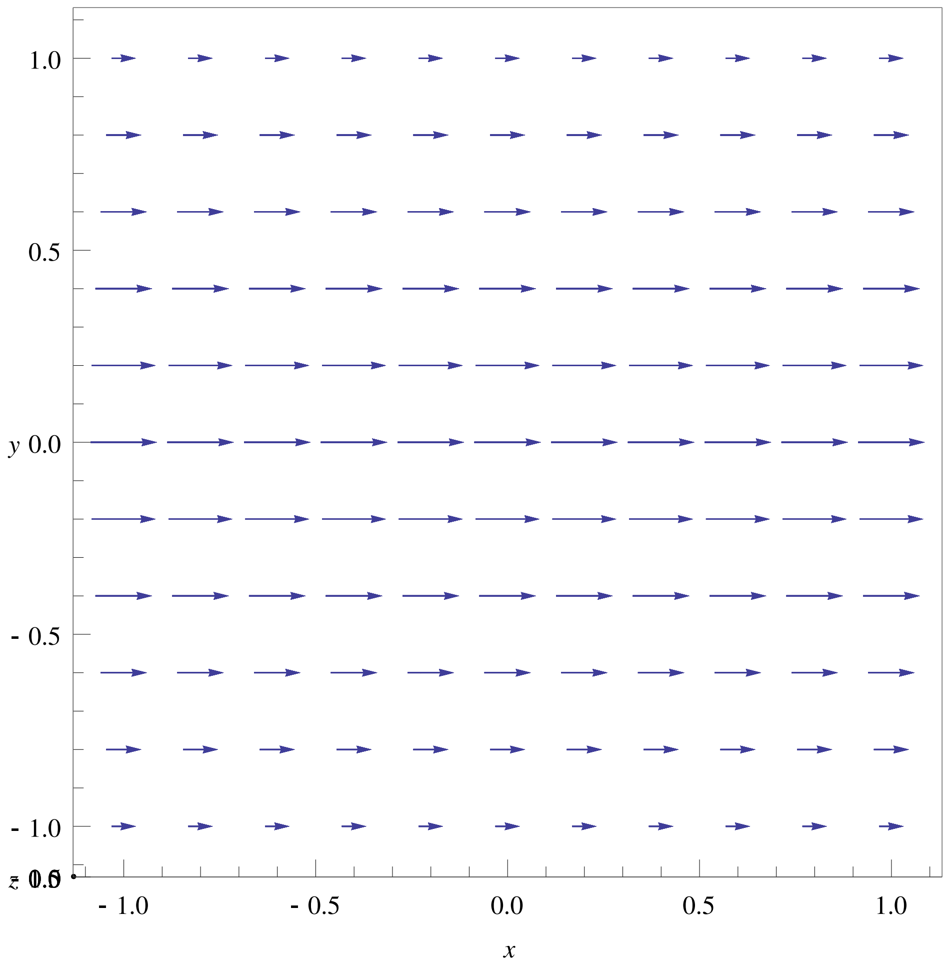
Student handout For each of the vector fields below, decide whether the divergence is postive, negative, or zero in each quadrant. Be prepared to defend your answers.



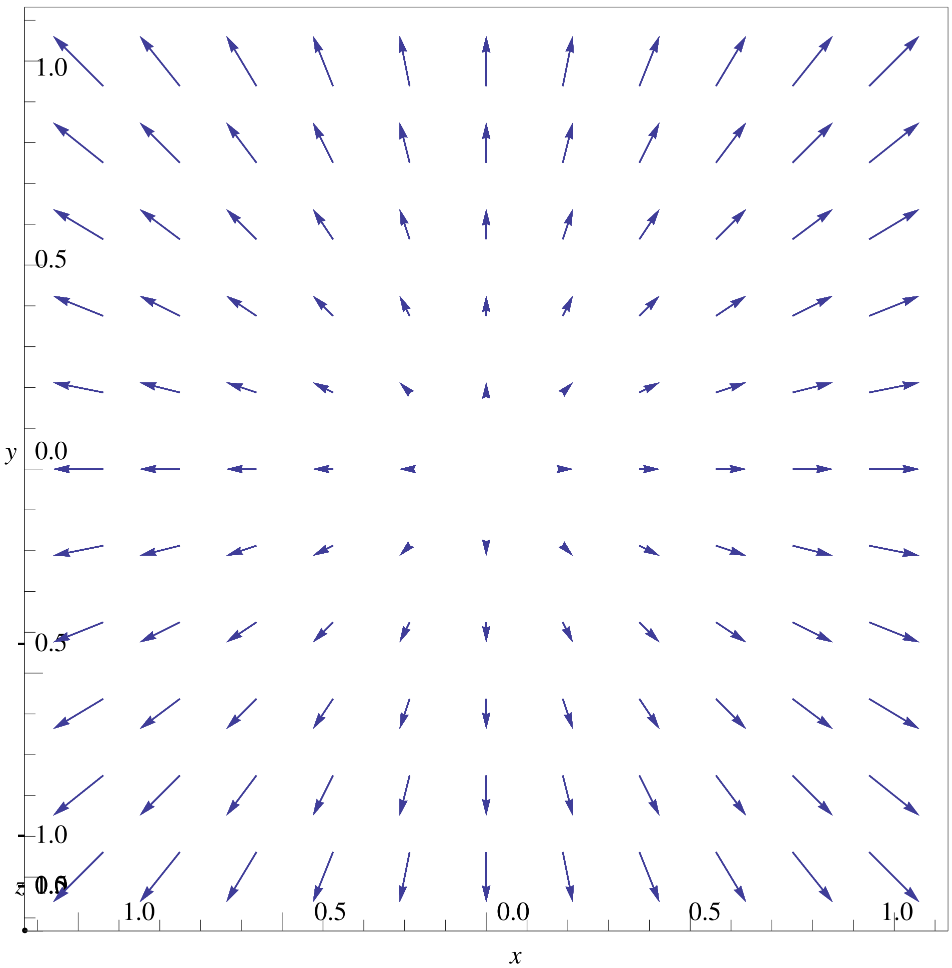
$$F_1$$



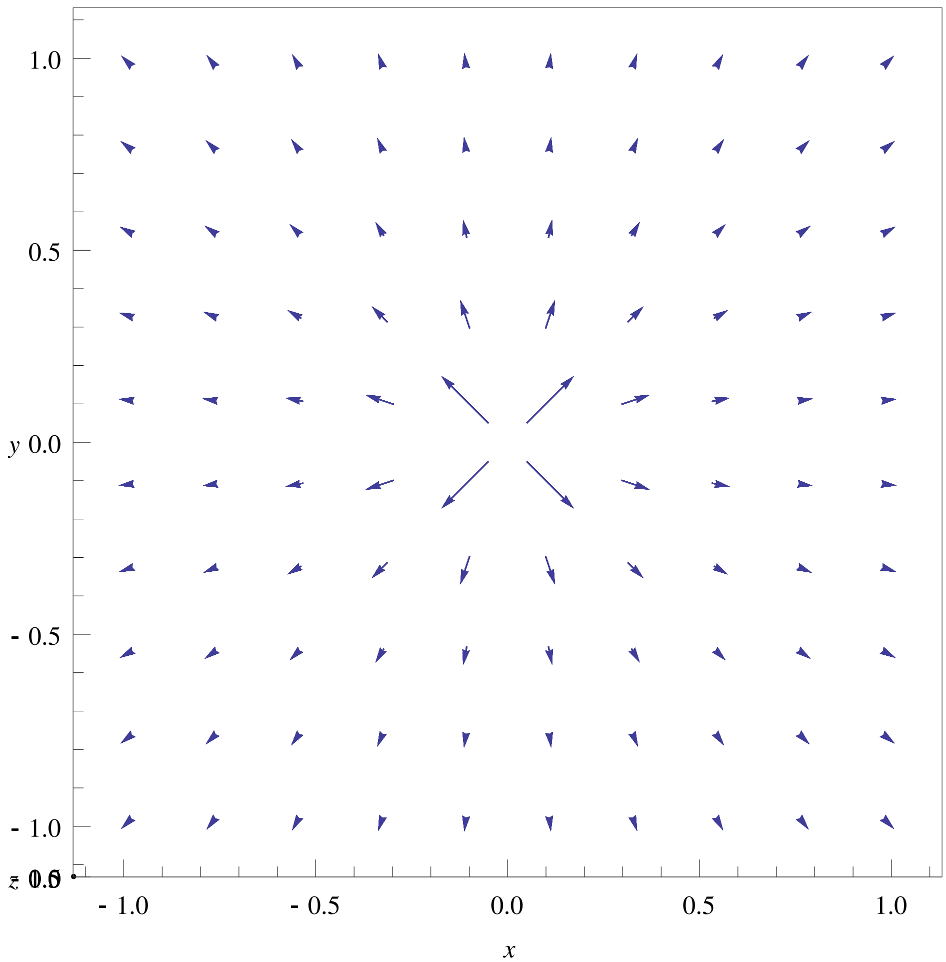
$$F_2$$



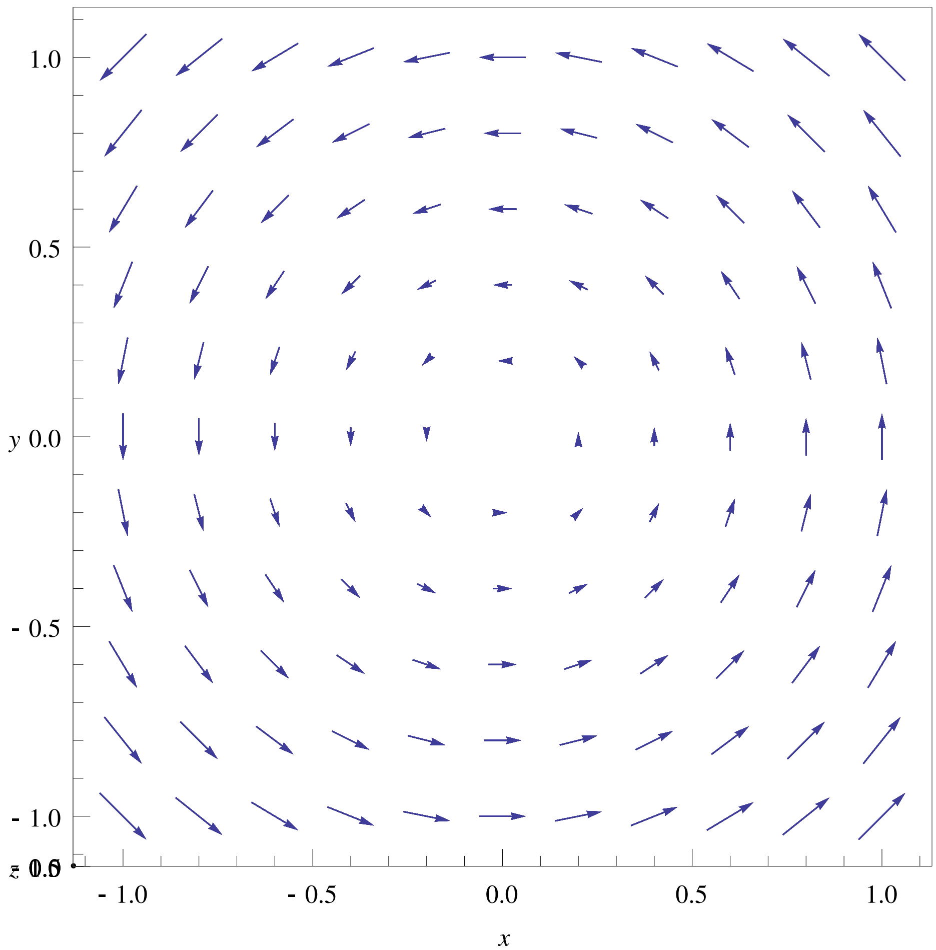
F_3



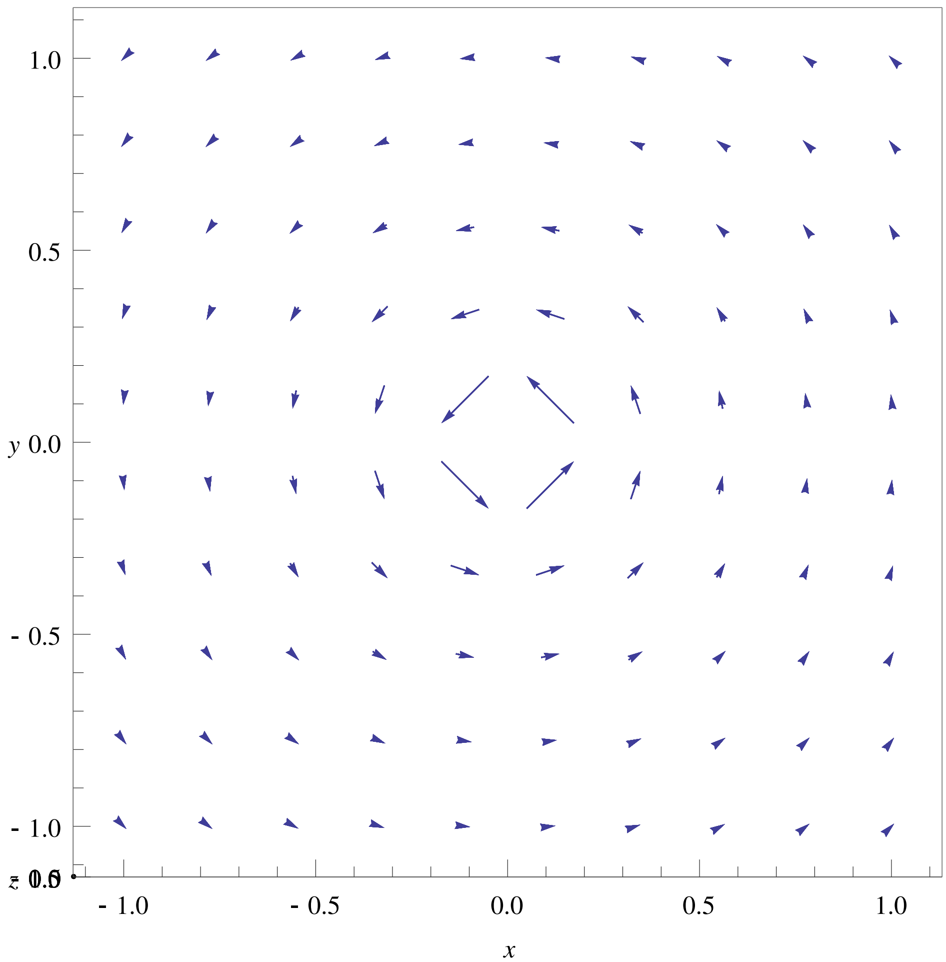
$$F_4$$



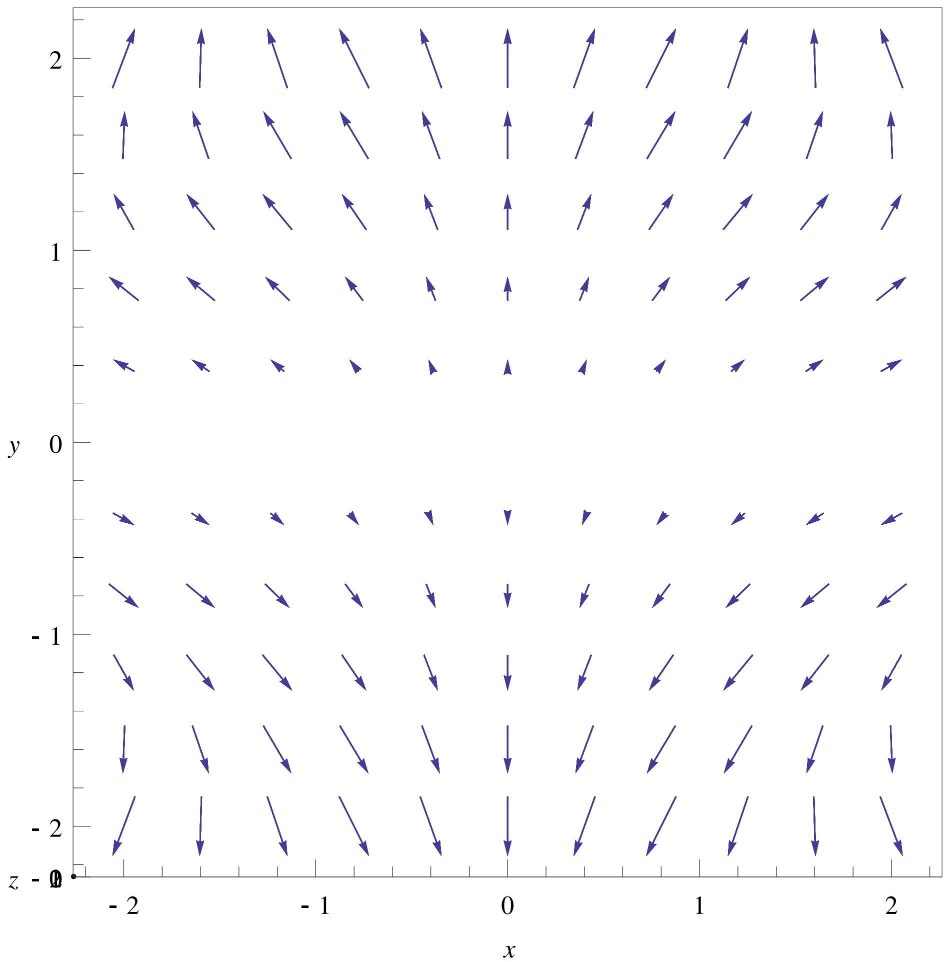
$$F_5$$



$$F_6$$



$$F_7$$



$$F_8$$

1 Instructor's Guide

1.1 Introduction

We precede this activity with a derivation of the rectangular coordinate expression for divergence: the divergence is the flux per unit volume through an appropriately chosen closed surface. Our derivation follows the one in *div grad curl and all that*, Schey, 2nd edition, Norton, 1973, p. 36 or see our versions. One can also use clicker questions or SWBQs about divergence to help get them started (see Visualization of Divergence).

Then the students are presented with a number of examples of 2-d vector fields in dry eraseable plastic sleeves. Most vector fields are shown as a cross-section of the field and it is assumed the the vector field is independent of the third (unshown) dimension. Students are asked to use the definition of divergence as the flux per unit volume through an infinitesimal box to predict the sign and relative magnitude of the divergence in each quadrant. Optionally, a prepared *Mathematica* worksheet can be used to calculate the divergence, so students can check their predictions.

1.2 Student Conversations

- To speed up this activity, the instructor can do the first example for the class.
- Many students are still confused about the role of the unit normal. Draw it!
- It can help to write the formula for the flux so that students are reminded that they are taking the dot product of the vector field with the unit normal.
- **Choosing the correct shape chunks:** Students should be encouraged to see that it is easier to choose a volume that respects the symmetries of the vector field, i.e. in this case, sides of pineapple chunks for the cylindrical fields.
- **Various points:** Make sure to look at several different points in space for each vector field, NOT just the origin. Use this to emphasize that divergence is itself a field.
- **Positive or negative divergence:** Students should should see that, for the vector fields that radiate out from the origin, different length scalings lead to different signs for the divergence, depending on whether they are adding larger vectors perpendicular to the larger arced surface or smaller vectors perpendicular to the larger arced surface. Near the end of this activity, they can be asked to discover which scaling leads to zero divergence everywhere (except at the origin). This vector field represents the electric field around a charged wire. Nature picks out this special case.

1.3 Wrap-up

A quick whole class discussion of the items listed in Student Conversations.