Changing Internal Energy: Consider a thermal system of water vapor where the only ways to change the internal energy are by heating or doing work:

$$\text{change in internal energy} = \text{heat into or out of system} + \text{work done on or by system}$$

For very small changes in internal energy, the above energy conservation statement turns into:

$$\begin{aligned}
\text{small change of internal energy} &= \text{small amount of heating} + \text{small amount of work done} \\
d\,\text{internal energy} &= (\text{temperature})\,d\,\text{entropy} - (\text{pressure})\,d\,\text{volume} \\
dU &= T\,dS - p\,dV
\end{aligned}$$

where "d quantity" indicates a very small change in a quantity.

Examine Your Intuition: For each of the three scenarios below, how would you expect the internal energy of the water vapor to change (increase, decrease, or stay the same)? Explain your reasoning.

> Instructor's guide **Alternative:** Assign different scenarios to different groups and run as a compare & contrast activity.

1. You have a lidded metal pot filled water vapor and put it on a hot stove.

> Instructor's guide **Answer:** Adding heat, no work—internal energy increases.
>
> **Discussion: No work done** Students need to recognize that putting the lid on means the volume is not changing → no work done. Most students have intuition that the heat transferred is not zero.
>
> **Discussion: Terminology** process is an isochor (fixed volume)
>
> **Just in Case:** Obstreperous physicists are sometimes concerned about the initial temperature of the water vapor – is it hotter than the stove? Tell these folks that the water vapor starts cooler than the stove.

2. You have an insulated piston (so that no heat enters or leaves) filled with water vapor and you push down on the lid.

> Instructor's guide **Answer:** Doing work on the system, no heating—internal energy increases.
>
> **Discussion: Work On the System** Students need to recognize that work is being done on the system (the volume is decreasing) so that the internal energy increases.
>
> **Discussion: Piston** Some students might not be familiar with what a piston is.
>
> **Discussion: Terminology** process is an adiabat/isentrope (no heat exchange and fixed entropy)

3. You have an uninsulated piston with a heavy top (which fixes the pressure) filled with water vapor and you put it on a hot stove.

> Instructor's guide **Discussion: Heat** A student might argue that heat instantly leaves the piston so that the internal energy doesn't change. As the metal piston heats up, it has to conduct heat to both the water vapor and the environment.
>
> **Discussion: Not enough information** A student might reasonably decide there is not enough information to determine the change in the internal energy – the change in internal energy is ambiguous without additional information.
>
> **Discussion: Terminology** process is an isobar (fixed pressure)

Interpret the Surface: The plastic surface model is a graph of the internal energy of water vapor as a function of volume and entropy. The arrows in the base of the surface indicate the direction of increase for each of these quantities. The height of the surface represents the value of the internal energy.

> Instructor's guide **Technical Detail:** The surface was created with data from the National Institute of Standards and Technology (NIST).
>
> **Intro:** You might choose to not hand out the surfaces until students finish the previous parts of the activity. It might be worth describing the features of the plastic surfaces to the class when you start handing out the surfaces or when groups get to this part in the activity.

For each of the scenarios described above, imagine that the water vapor starts with values of volume and entropy that correspond to the blue dot. For each scenario:

1. What does the surface tell you about the change in internal energy? Is this consistent with your intuition?

> Instructor's guide **Scenario 3:** Because the surface has the equations of state baked into it, there is no ambiguity about the change in internal energy for Scenario 3 - the internal energy increases.

2. Describe in words how you determined this information from the surface.

> Instructor's guide **Answer:** Find the change by following the appropriate path. On the Purple surface, constant volume runs parallel to the Entropy axis—U increases. Constant entropy is parallel to the Volume axis—U decreases. Constant pressure goes along Pressure contours—U increases a bit.

3. Would the change in internal energy be different if you started somewhere other than the blue dot?

> Instructor's guide **Answer:** The surface is not a plane, so the amount of change in internal energy will depend on where you start. The relationships between variables are monotonic, so the **sign** of the change (whether an increase or decrease) does not depend on where you start.

> Instructor's guide Modifications for remote instruction using a the contour map:
> **Interpret the Graph:** The graph shows internal energy contours plotted on volume and entropy axes. One pressure contour is also plotted.
>
> For each of the scenarios described above, imagine that the water vapor starts with values of volume and entropy that correspond to the blue dot. For each scenario:
>
> 1. What does the graph tell you about the change in internal energy? Is this consistent with your intuition?
> 2. Describe in words how you determined this information from the graph.
> 3. Would the change in internal energy be different if you started somewhere other than the blue dot?
>
> **Answer:** The surface is not a plane, so the amount of change in internal energy will depend on where you start.
>
> **Discussion: Monotonicity** A great follow-up question is "Does the sign of the change depend on where you start? How can you tell?" The relationships between variables are monotonic, so the **sign** of the change (whether an increase or decrease) does not depend on where you start.
>
> Monotonicity is a general feature of thermal systems.

Instructor's guide

SUMMARY PAGE

Goals:

- Both heat and work contribute to changing internal energy.
- Reinforces that: working corresponds to volume changing (if volume is constant, no work is done). Heating means entropy is changing (if entropy is constant, no heat is transferred).
- Thermal systems follow paths in state space—different quasistatic processes follow different paths.
- The internal energy of the system can change in various ways (increase, decrease, stay the same)—the change varies by process.
- The amount of change generally depends on the initial state.

Time Estimate: 30 minutes

Tools:

- Purple $U(S, V)$ Plastic Surface Graph
- For Remote Option: Changes in Internal Energy contour maps
- Student handout for each student
- A personal or shared writing space for each student to write/draw/sketch.

Intro:

- **Option:** You can start with a brief introduction to the 1st law of thermodynamics (as in the "Changing Internal Energy" section). That information could be displayed to the whole class on a board or screen and then removed from the worksheet.
- **Option:** Some students really want to know what entropy is—knowing that adding/removing heat corresponds to changes in entropy is useful.
- **Option:** If students are not familiar with what a "physics piston" looks like, you could show them a diagram.
- Students will need some orientation to the surfaces when you distribute them.
  - The height is the internal energy.
  - The independent variables of the purple surface are volume and entropy – alert the students to the embossed axes on the base.
  - The etched contours are lines of constant temperature and pressure. The temperature contours are (nearly) parallel to internal energy level curves (i.e., parallel to the base).

    (Students who are familiar with the fact that for an ideal gas U=NkT might recognize that T and U have the same level curves. Water vapor is not quite an ideal gas.)

To be productive in this activity, students should be familiar with:

- That a piston is a container with a movable lid that changes volume. In contrast, a lidded pot has a fixed volume.
- That work done is related to changes in volume and heating is related to changes in entropy.
- That heat and temperature are not the same. Heating is related to changes in entropy.

Whole Class Discussion:

- "Did all of these processes correspond to the same change in internal energy?"

  "How could you tell from the surface?"

  "How was this related to your intuition?"

- "Would you get the same result if you started at the red star?"

Option: Students can compare the change in internal energy for scenario #2 and #3 for equal steps in entropy—for the isobar, U changes less because some of the energy from heating goes into doing work on the environment.

Option: "What experiment could I do to keep the temperature of the system constant? How would the internal energy change in that scenario?"