Instructor's guide **What Students Learn:**

- The choice of where $U = 0$ is arbitrary and doesn't affect the difference in potential energy between two locations.
- The zero of gravitational potential energy is conventionally set at $r \to \infty$, making the potential energy negative, $U < 0$. "Increasing" potential energy approaches $U = 0$.
- $U = -GMm/r$, is valid outside spherically symmetric Earth, but not inside it.
- The Earth-satellite system can be idealized as spherically symmetric. This prepares students for later discussions about equipotentials and about radial symmetry.

Time Estimate: 30 minutes + 20 minute class discussion.
Equipment:

- Green "sphere" surface for each group
- Dry-erase markers & erasers
- Whiteboard for each group
- Student handout for each student

Prerequisites:

- This activity works particularly well when done **before** the *Gravitational Force* activity, even if you introduce force before gravitational potential energy, because students examine how the surface represents the physical system.
- Students should know about potential energy but do not need to know about universal gravitation before the activity. The instructor will introduce the equation $U = -GMm/r$ during this activity.

Introduction:

- Explain that the
  - center of Earth is in the lowest corner,
  - the etched line represents the surface of Earth,
  - the base is the xy-plane, and
  - the height of the surface represents the gravitational potential energy for locations in the xy-plane.

Whole Class Discussion / Wrap Up:

- There are various strategies for measuring heights on the surface
- Differences in energy are what matter, not the values
- Spherical symmetry means we can use r to fully describe the energy

Your group has a plastic surface that represents the gravitational potential energy of a space station-Earth system as a function of the position of the space station relative to Earth. The height of the surface corresponds to the value of gravitational potential energy.

Interpret the Surface:

- Mark a point on the surface where the gravitational potential energy of the system is zero.

> Instructor's guide **Student Ideas:** all are ok.
> - the center of Earth,
> - the surface of Earth,
> - the blue circle,
> - some arbitrary point,
> - the far corner of the surface to indicate infinitely far away

- If a one inch difference of height corresponds to an energy difference of 1 TJ, what is the gravitational potential energy of the system when the space station is at the blue circle?

> Instructor's guide **Student Ideas:** Students need to measure the difference in height between the blue circle and their zero.
> - hold ruler vertically and align estimate vertical displacement between zero location and blue circle
> - measure height from base to zero and from base to blue circle
> - turn the surface over and measure depths
> - Few students will measure the shortest-path distance between their zero and the blue circle.

- What is the difference in gravitational potential energy of the system if the space station moves from the blue circle to the red star?

> Instructor's guide **Answer:** All groups should see the potential energy increase of +0.5 TJ
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> As a short class discussion, have the students help you build a table for the two measured values. One column will vary, the other column should be approximately +0.5 TJ for all groups.

Find Patterns: For each of the locations listed below, identify all other points on the plastic surface model with the same gravitational potential energy:

(a) the blue circle

(b) the orange star

(c) the green square

(d) the point you marked in question 1.

What patterns are you noticing?

> Instructor's guide **Nudge:** "If the space station was at the blue circle, how could it move so that the gravitational potential energy doesn't change?"
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> **Whole Class Discussion:** "What patterns did your group notice?" (Give students time to discuss within their group then let groups report out)
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> Possible patterns:
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> - circles/arcs
> - horizontal spacing for circle/star/square increasing
> - vertical spacing for circle/star/square the same

Describe an Orbit: Draw a dot on your whiteboard to represent the center of Earth. Draw a closed, elliptical orbit of the space station. Using the information in the plastic surface model, what happens to the gravitational potential energy as it moves around the orbit?

If the space station does not use its engines during the orbit so that the total energy is constant, what happens to the speed of the space station?

> Instructor's guide **Nudge:** To simplify, ask students about the earlier case of a circular orbit.
> **Student Ideas:** The speed will change. Conservation of energy says that if the space station is closer to earth, its speed must increase.
> **Extension:** Draw an orbit so that the space station is always over the same spot on the Earth as it rotates (a *geostationary* orbit).

Plot: Using your whole whiteboard, sketch a large graph that shows how the gravitational potential energy of the system depends on the space station's distance from the center of Earth. Clearly label the axes of your graph. Be ready to show where you chose $U = 0$ and to explain your reasoning to your classmates.

> Instructor's guide **Extension:** If a group gets done early, ask them to mark where the blue circle, orange star, and green square are on your plots.
> **Extension:** Remind students that GPE is a function of three spatial variables. Ask them to describe the equipotentials (spherical shells) or the (spherical) symmetry involved.

> Instructor's guide **Interpret the Surface**
> **Purpose:** We want students to physically manipulate the surface so that they kinesthetically understand the surface height as representing gravitational potential energy. We also want students to learn that although the choice of zero changes the value of the gravitational potential energy at any point, the difference in energy between two points will always be the same. In this situation, the *difference* is the physically meaningful quantity.
> **Discussion:** Students must be ***told*** that the height of the surface represents GPE, and that one inch of height is a change of 1 TJ of energy. The goal of this task is for students to reinforce the idea of height *difference* as the meaningful measurement. For students who are stuck on making the measurement, you can guide them with a question like "If the space station is initially at the blue dot, how could the space station move so that the gravitational potential energy doesn't change?" Once they recognize that the equipotentials are circles, they may see that there is a point on the edge where they can measure the value of GPE. A discussion of the variety of measurement methods can happen in the whole class discussion: Students might place the ruler at the base or highest point and visually see where the blue dot falls, or they might place the ruler on the dot (perhaps turning the surface upside down) and visually see where the edge, or the highest/lowest points fall. They may draw on the surface to extend to one or both edges, using a straight line (incorrect) or the circular equipotentials. One note in all of this is that the lowest point on the plastic graph is higher than the base.
> **Find Patterns**
> **Purpose:** Students are being prompted here to draw equipotentials, which in this case appear as quarter circles on the surface. The prompt says "points" which may be confusing to some students. You are welcome to clarify that there may be *many* points.
> **Goal:** The goal is for students to ***discover*** patterns and foreshadow concepts for later discussions. There is a lot to talk about regarding the relationship between the equipotentials and the equation, but keep in mind that the equation will be introduced later. So right now we just want students to identify the qualitative features of the equipotentials. Possible patterns:
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> - circles/arcs [spherical symmetry - what do these circles have to do with symmetry?]
> - horizontal spacing for circle/star/square increasing [foreshadowing inverse law - what would the next point be? (Don't force it if students don't seem ready)]
> - vertical spacing for circle/star/square the same [equal differences in potential. This is the convention for equipotentials.].

Describe an Orbit Students may need to be ***told*** that elliptical orbits are possible. The goal of this task is for students to explore the physics of elliptical orbits and ***discover*** that objects in an elliptical orbit will change speed.
Plot The whole class discussion is an interactive opportunity for students to explore the limitations of the mathematical model and investigate the meaning and conventions around the zero of gravitational potential.

- Why choose $U(r \to \infty) = 0$? To make the equation simpler - a different choice is just as correct! The conventional choice means that the gravitational potential energy is negative, which has no special physical meaning. Be careful about language - when potential energy "decreases", it gets more negative (like temperature, not like velocity where "decrease" means "smaller magnitude").
- When can you use the equation $U = -GMm/r$? This equation describes GPE well for regions outside the surface of Earth, but inside the GPE is a different function of r. $U = -GMm/r$ is the gravitational potential energy between two point masses. We can treat Earth as a point mass (but only for regions outside the Earth's surface) because of the spherical symmetry of how the mass is distributed.

Following is a detailed description of how this discussion might be facilitated.

(a) Begin by having students arrange themselves so they can all see each others' whiteboards. Give them a full 30 seconds to study the graphs.

(b) Students were prompted to attend to where zero is on their graph, and ensure their graph is consistent with where they picked zero at the beginning of the activity. Call on a few representative groups to describe their graphs. Different groups may have chosen different locations as the "zero" for GPE—having groups share what they chose and why can help emphasize the freedom to choose where zero is located. We suggest calling on group that used the conventional zero at $r = \infty$ somewhere in the middle of these presentations, and encouraging the other choices, so students know their choice is equally valid to any other. (e.g."I think it makes a lot of sense to place zero at the blue dot, since that is where the space station is located") This will help students to not feel "wrong" when discussing the merits of the conventional choice.

(c) Students are not expected to know or discover the equation for universal gravitation. They must be ***told*** this information. You can work with the students to determine the $-\frac{1}{r}$ dependence of the graph OR just cut to the chase and display the following equation to the students: "$U_{grav} = -G\frac{Mm}{r}$". Tell students "Without erasing the graph on your whiteboard, add a graph of this equation." Prompt the students to return to their groups and work on this.

(d) Spend a few minutes observing the groups. If a group set their $U = 0$ somewhere other than $r = \infty$ ask, "How could you adjust this equation so it fits your graph better?" (Answer: By adding a constant shift to the equation: $U_{grav} = -G\frac{Mm}{r} + B$.) Some groups may neglect the negative sign, and should be reminded that this flips the graph.

(e) Gather the groups together again. Ask a few groups (that may have not shared out yet) what they did to adjust the fit. Goal of discussion: With the addition of a constant shift up or down, the equation fits well for $r > R_{earth}$, and but is still not a good description of the potential for $r < R_{earth}$.

(f) Show the students the attached graph. This can be concluded in a variety of ways, depending on student interest:
   - Highlight the fact that, although we don't have an equation for this system, the graph of U can be determined from the surface itself.
   - Ask the class, "Why is it reasonable to represent the system as a graph with only 2 axes (U and r)?"
   - Students can be ***told*** what the *conventional* choice for $U = 0$ is and why this convention is chosen.
   - Have students identify where the zero of this graph is found on the surface.
   - Ask the class, "What does it mean to say that a system has negative potential energy?"
   - Compare the two types of questions that you can ask about potential energy: (1) questions where you must know where zero is ("What is U at this point?") and (2) questions where you don't need to know the offset ("What is the change in potential between these two points? What is the derivative at this point?")

- Ask students to speculate about why the $1/r$ model does not work inside the earth. (The equation can be found by integrated sources or by integrating the gravitational field found by Gauss' Law.)
- A discussion of energy diagrams follows easily from this activity.
- Initiate a discussion about the rings they drew, and define what an equipotential is.
- This model only considers the space station-earth system. What would happen if we wanted to also consider the sun?
- Despite the choice of where $U = 0$ affecting the sign and value of the GPE at any particular point, the idea of "increasing" and "decreasing" still have reasonable meaning.