Student handout

Block Sliding Down a Frictionless Wedge

(Taylor Example 7.5) Consider a block with mass m sliding frictionlessly down an wedge with mass M that makes an angle α. The wedge itself slides frictionlessly across a horizontal floor near the surface of Earth. The block is released from the top of the wedge, with both objects initially at rest.

If length of the sloping face is d, how long does the block take to reach the bottom?

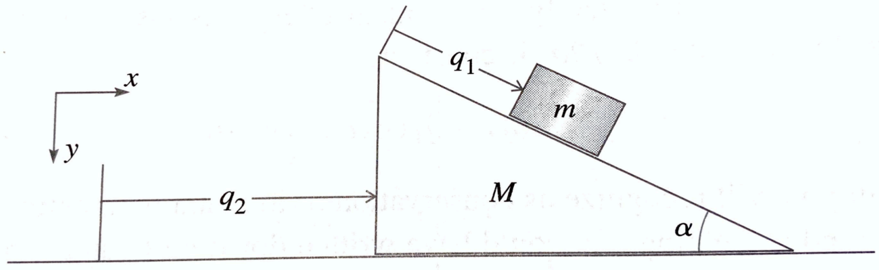

Figure 7.8 A block of mass m slides down a wedge of mass M, which is free to slide over the horizontal table.

1 Structure of the Activity

This is a small group activity where everyone is solving the same problem. The problem is hard, so I break the problem up for the students as we go along. Students are asked to share their reasoning with the whole class.

1. Describe the problem situation, emphasizing the choice of coordinates. Distribute the handout. Tell students they are going to first solve for the acceleration in order to then calculate the time.

2. Ask students to work in groups for 5-10 min. to anticipate what the accelerations will look like:

   - Dimensions
   - Functional Behavior
   - Special Cases (demonstrate one, like the case where the wedge is flat, i.e., $\alpha = 0$).

   They will use these anticipations to make sense of their answers.

3. Bring the whole class together and make a list of the anticipations. Ask students to justify their anticipations.

4. Ask students to work in groups to find the accelerations. They will get stuck writing down the kinetic energy. Let them think about it for a little bit but bring the whole class together to talk it through.

5. Once you've written the Lagrangian down as a class, ask students to work in groups to solve for the accelerations.

6. Solve for the accelerations as a whole class and then talk thought comparing the anticipations with the calculations.

2 Student Conversations

- **Using a Non-Orthogonal Coordinate System** Some students believe that coordinate systems have to be orthogonal. The question "How can you use a coordinate system like this?" can be responded to with, "Why would you not be able to?" Let them think for a minute, then mention: you do have to be careful with dot products - they're not zero.

  Some students are also bothered with coordinate systems where the zeroes don't line up.

  Some students are further still bothered by the fact that the zero of q_1 moves in space because it is attached to the top of the wedge. Relatedly, some students are bothered that the speed of the wedge is independent of q_1. Some students might notice that this is a non-inertial coordinate system. If you've discussed relativity already, it can help to talk about different reference frames.

  It is helpful to tell students that it's ok for this to feel uncomfortable because its unfamiliar - they haven't really seen anything like this! But this coordinate system is perfectly reasonable. Examples they've previously worked with: they've rotated a Cartesian coordinate system to be parallel and perpendicular to an incline. For problems with rotational and translational motion, they might use Cartesian to describe the position of the center of mass and polar to describe the rotational motion about the center of mass - two coordinate systems with different origins for the same problem.

- **Labeling Different Speeds as Different:** The box and the wedge will have different speeds. They should be labeled differently.

- **Special Cases:** Students have a really hard time anticipating the special cases and then interpreting the cases once they get the answer, particularly the case where $m >> M$. Plan on spending a good amount of class time on this discussion.