

Student handout

Electrostatic Potential from Two Charges

- Find a formula for the electrostatic potential $V(\vec{r})$ that is valid everywhere in space for the following two physical situations:
 - Two charges $+Q$ and $+Q$ placed on a line at $z' = D$ and $z'' = -D$.
 - Two charges $+Q$ and $-Q$ placed on a line at $z' = D$ and $z'' = -D$, respectively.
- Simplify your formulas in the limiting cases of:
 - the x -axis
 - the z -axis
- Discuss the relationship between the symmetries of the physical situations and the symmetries of the functions in these limiting cases.

1 Instructor's Guide

1.1 Introduction

It may help to do the The Distance Formula (Star Trek) activity before this one. There is an alternative version of this activity Electrostatic Potential Due to a Pair of Charges (with Series) in which students find series expansions of the potential along the axes of symmetry.

Students typically know the iconic formula for the electrostatic potential of a point charge $V = \frac{kq}{r}$. We begin this activity with a short lecture/discussion that generalizes this formula in a coordinate independent way to the situation where the source is moved away from the origin to the point \vec{r}' , $V(\vec{r}) = \frac{kq}{|\vec{r} - \vec{r}'|}$. (A nice warm-up (SWBQ) to lead off the discussion:

Introductory SWBQ Prompt: “Write down the electrostatic potential everywhere in space due to a point charge that is not at the origin.” The lecture should also review the superposition principle.

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \sum_i \frac{q_i}{|\vec{r} - \vec{r}'_i|}$$

This general, coordinate-independent formula should be left on the board for students to consult as they do this activity.

1.2 Student Conversations

- Students create an expression such as

$$V(x, y, z) = \frac{Q}{4\pi\epsilon_0} \left(\frac{1}{|z - D|} + \frac{1}{|z + D|} \right)$$

Each axis and charge distribution has a slightly different formula. A few groups may have trouble coordinatizing $|\vec{r} - \vec{r}'|$ into an expression in rectangular coordinates, but because the coordinate system is set up for them, most students are successful with this part fairly quickly.

- Many students are likely to treat this as a two-dimensional case from the start, ignoring the y axis entirely. Look for expressions like

$$V = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^N \frac{q_i}{\sqrt{x^2 + (z - z_i)^2}}$$

Prompt them to consider the larger, 3-dimensional picture.

- Most students will leave off the absolute value signs when evaluating the potential on the z -axis. If they do this, their formulas will not be correct for negative values of z . The subtlety here is that

$$\sqrt{a^2} = |a|$$

not

$$\sqrt{a^2} = a$$

in contexts like this when $\sqrt{a^2}$ is a distance and therefore necessarily positive and when a itself might be either positive or negative.