

## Student handout

**1 1-D Particle-in-a-box**

Eigenstates:

$$|n\rangle \doteq \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L} \quad (1)$$

$$n = \{1, 2, 3, \dots\} \quad (2)$$

Eigenvalue Equations:

$$\hat{H} |n\rangle = \frac{\pi^2 \hbar^2}{2\mu L^2} n^2 |n\rangle \quad (3)$$

$$(4)$$

**2 Particle-on-a-Ring**

Eigenstates:

$$|m\rangle \doteq \frac{1}{\sqrt{2\pi r_0}} e^{im\phi} \quad (5)$$

$$m = \{\dots, 2, 1, 0, -1, -2, \dots\} \quad (6)$$

Eigenvalue Equations:

$$\hat{H} |m\rangle = \frac{\hbar^2}{2I} m^2 |m\rangle \quad (7)$$

$$\hat{L}^2 |m\rangle = \hbar^2 m^2 |m\rangle \quad (8)$$

$$\hat{L}_z |m\rangle = \hbar m |m\rangle \quad (9)$$

**3 1-D Harmonic Oscillator**

Eigenstates:

$$|n\rangle \doteq \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} \frac{1}{\sqrt{2^n n!}} H_n(\xi) e^{-\xi^2/2} \quad (10)$$

$$\xi = \sqrt{\frac{m\omega}{\hbar}} x \quad (11)$$

$$n = \{0, 1, 2, 3, \dots\} \quad (12)$$

Eigenvalue Equations:

$$\hat{H} |n\rangle = \hbar\omega \left( n + \frac{1}{2} \right) |n\rangle \quad (13)$$

$$(14)$$

## 4 2-D Particle-in-a-Box

Eigenstates:

$$|mn\rangle \doteq \sqrt{\frac{2}{L_x}} \sqrt{\frac{2}{L_y}} \sin \frac{m\pi x}{L_x} \sin \frac{n\pi y}{L_y} \quad (15)$$

$$m = \{1, 2, 3, \dots\} \quad (16)$$

$$n = \{1, 2, 3, \dots\} \quad (17)$$

Eigenvalue Equations:

$$\hat{H} |mn\rangle = \frac{\pi^2 \hbar^2}{2\mu} \left( \frac{m^2}{L_x^2} + \frac{n^2}{L_y^2} \right) |mn\rangle \quad (18)$$

$$(19)$$

## 5 Particle-on-a-Sphere

Eigenstates:

$$|\ell m\rangle \doteq Y_\ell^m(\theta, \phi) \quad (20)$$

$$= (-1)^{\frac{m+|m|}{2}} \sqrt{\frac{2\ell+1}{4\pi} \frac{(\ell-m)!}{(\ell+m)!}} P_\ell^m(\cos \theta) e^{im\phi} \quad (21)$$

$$\ell = \{0, 1, 2, \dots\} \quad (22)$$

$$m = \{\ell, \dots, 0, \dots, -\ell\} \quad (23)$$

Eigenvalue Equations:

$$\hat{H} |\ell m\rangle = \frac{\hbar^2}{2I} \ell(\ell+1) |\ell m\rangle \quad (24)$$

$$\hat{L}^2 |\ell m\rangle = \hbar^2 \ell(\ell+1) |\ell m\rangle \quad (25)$$

$$\hat{L}_z |\ell m\rangle = \hbar m |\ell m\rangle \quad (26)$$

## 6 3-D Particle-in-a-Box

Eigenstates:

$$|mnp\rangle \doteq \sqrt{\frac{2}{L_x}} \sqrt{\frac{2}{L_y}} \sqrt{\frac{2}{L_z}} \sin \frac{m\pi x}{L_x} \sin \frac{n\pi y}{L_y} \sin \frac{p\pi z}{L_z} \quad (27)$$

$$m = \{1, 2, 3, \dots\} \quad (28)$$

$$n = \{1, 2, 3, \dots\} \quad (29)$$

$$p = \{1, 2, 3, \dots\} \quad (30)$$

Eigenvalue Equations:

$$\hat{H} |mnp\rangle = \frac{\pi^2 \hbar^2}{2\mu} \left( \frac{m^2}{L_x^2} + \frac{n^2}{L_y^2} + \frac{p^2}{L_z^2} \right) |mnp\rangle \quad (31)$$

$$(32)$$

## 7 Hydrogen Atom

Eigenstates:

$$|n\ell m\rangle \doteq R_{n\ell}(r) Y_\ell^m(\theta, \phi) \quad (33)$$

$$= -\sqrt{\left(\frac{2Z}{na_0}\right)^3 \frac{(n-\ell-1)!}{2n[(n+\ell)!]^3}} \left(\frac{2\rho}{n}\right)^\ell e^{-\frac{\rho}{n}} L_{n+\ell}^{2\ell+1}\left(\frac{2\rho}{n}\right) (-1)^{\frac{m+|m|}{2}} \sqrt{\frac{2\ell+1}{4\pi} \frac{(\ell-m)!}{(\ell+m)!}} P_\ell^m(\cos\theta) e^{im\phi} \quad (34)$$

$$\rho = \frac{Zr}{a_0} \quad (35)$$

$$n = \{1, 2, 3, \dots\} \quad (36)$$

$$\ell = \{0, 1, 2, \dots, n-1\} \quad (37)$$

$$m = \{\ell, \dots, 0, \dots, -\ell\} \quad (38)$$

Eigenvalue Equations:

$$\hat{H} |n\ell m\rangle = -\frac{1}{2} \left( \frac{Ze^2}{4\pi\epsilon_0} \right)^2 \frac{\mu}{\hbar^2} \frac{1}{n^2} |n\ell m\rangle \quad (39)$$

$$= -13.6\text{eV} \frac{1}{n^2} |n\ell m\rangle \quad (40)$$

$$\hat{L}^2 |n\ell m\rangle = \hbar^2 \ell(\ell+1) |n\ell m\rangle \quad (41)$$

$$\hat{L}_z |n\ell m\rangle = \hbar m |n\ell m\rangle \quad (42)$$