There are two major strategies to find these coefficents we could use, we will start with the more general one first.

In order to find the coefficents we want, we need to do an inner product like so:

$$c_{\ell,m} = \langle \ell, m | Y(\theta, \phi) \rangle \tag{1}$$

Since we were given a wavefunction, we need to interpret this inner product as an integral over all space in order to use our wavefunction as is:

$$c_{\ell,m} = \int_0^{2\pi} \int_0^{\pi} Y_\ell^{*m} Y(\theta, \phi) \sin(\theta) d\theta d\phi \tag{2}$$

Find the non-zero coefficients for an expansion in spherical harmonics of the function:

$$Y(\theta, \phi) = \left(\frac{15}{16\pi} \right)^{\frac{1}{2}} \sin(2\theta) \sin(\phi)$$