Outer Product of a Vector on Itself

1. For one of the vectors below, what matrix is the outer product of the vector on itself (i.e., $|v_1\rangle \langle v_1|$)? All the vectors are written in the S_z basis.

$$|+\rangle \doteq \begin{bmatrix}1\\0\end{bmatrix} \qquad |-\rangle \doteq \begin{bmatrix}0\\1\end{bmatrix} \qquad |+\rangle_x \doteq \frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix}$$

$$|-\rangle_x \doteq \frac{1}{\sqrt{2}}\begin{bmatrix}1\\-1\end{bmatrix} \qquad |+\rangle_y \doteq \frac{1}{\sqrt{2}}\begin{bmatrix}1\\i\end{bmatrix} \qquad |-\rangle_y \doteq \frac{1}{\sqrt{2}}\begin{bmatrix}1\\-i\end{bmatrix}$$

$$|v_7\rangle \doteq \frac{1}{5}\begin{bmatrix}3\\4\end{bmatrix} \qquad |v_8\rangle \doteq \frac{1}{5}\begin{bmatrix}4\\-3\end{bmatrix} \qquad |v_9\rangle \doteq \begin{bmatrix}a\\be^{i\phi}\end{bmatrix}$$

$$|1\rangle_x \doteq \frac{1}{\sqrt{2}}\begin{bmatrix}\frac{1}{\sqrt{2}}\\1\\\frac{1}{\sqrt{2}}\end{bmatrix} \qquad |0\rangle_x \doteq \frac{1}{\sqrt{2}}\begin{bmatrix}1\\0\\-1\end{bmatrix} \qquad |-1\rangle_x \doteq \frac{1}{\sqrt{2}}\begin{bmatrix}\frac{1}{\sqrt{2}}\\-1\\\frac{1}{\sqrt{2}}\end{bmatrix}$$

2. What is the transformation caused by your outer product?
3. What is the determinant of your outer product?
4. What is the square of your outer product?

   Bonus: What happens when you add the outer products for a complete orthonormal basis?

   Bonus 2: How would you answer questions (2) and (4) staying purely in Dirac bra-ket notation?