Equations for any 1-D System with Time-Independent Hamiltonian

Expansion in an Eigenbasis:

$$|\psi\rangle = \sum_m c_m |m\rangle \tag{1}$$

$$c_m = \langle m|\psi\rangle \tag{2}$$

$$\psi(x) = \langle x|\psi\rangle \tag{3}$$

Probabilities:

$$\mathcal{P}_m = |c_m|^2 \tag{4}$$

$$= |\langle m|\psi\rangle|^2 \tag{5}$$

$$\langle \hat{A}\rangle = \langle\psi|\hat{A}|\psi\rangle \tag{6}$$

$$= \sum_m a_m \mathcal{P}_m \tag{7}$$

$$\Delta\hat{A} = \sqrt{\langle\hat{A}^2\rangle - \langle\hat{A}\rangle^2} \tag{8}$$

Projection Postulate:

$$|\psi'\rangle = \frac{P_m|\psi\rangle}{\sqrt{\langle\psi|P_m|\psi\rangle}} \tag{9}$$

$$P_m = |m\rangle\langle m| \tag{10}$$

Time Dependence:

$$i\hbar\frac{\partial}{\partial t}|\psi\rangle = H|\psi\rangle \tag{11}$$

$$|\psi(t)\rangle = \sum_m c_m |m\rangle e^{-i\frac{E_m}{\hbar}t} \tag{12}$$

Equations Specific to the Ring

Operators and Eigenvalue Equations:

$$L_z \doteq -i\hbar\frac{\partial}{\partial\phi} \tag{13}$$

$$H \doteq -\frac{\hbar^2}{2I}\frac{\partial^2}{\partial\phi^2} \tag{14}$$

$$[H, L_z] = 0 \tag{15}$$

$$L_z|m\rangle = m\hbar|m\rangle \tag{16}$$

$$H|m\rangle = E_m|m\rangle \tag{17}$$

$$= \frac{\hbar^2}{2I}m^2|m\rangle \tag{18}$$

$$\tag{19}$$

Eigenfunctions:

$$|m\rangle \doteq \frac{1}{\sqrt{2\pi r_0}} e^{im\phi} \tag{20}$$

$$m = \{-\infty, \ldots, -1, 0, 1, \ldots, \infty\} \tag{21}$$

Inner Product:

$$\langle \psi_1 | \psi_2 \rangle = \int_0^{2\pi} \psi_1^*(\phi)\, \psi_2(\phi)\, r_0 d\phi \tag{22}$$