1 1-D Particle-in-a-box

Eigenstates:

$$|n\rangle \doteq \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L} \tag{1}$$

$$n = \{1, 2, 3, \dots\} \tag{2}$$

Eigenvalue Equations:

$$\hat{H} |n\rangle = \frac{\pi^2 \hbar^2}{2\mu L^2} n^2 |n\rangle \tag{3}$$

$$\tag{4}$$

2 Particle-on-a-Ring

Eigenstates:

$$|m\rangle \doteq \frac{1}{\sqrt{2\pi r_0}} e^{im\phi} \tag{5}$$

$$m = \{\dots 2, 1, 0, -1, -2, \dots\} \tag{6}$$

Eigenvalue Equations:

$$\hat{H} |m\rangle = \frac{\hbar^2}{2I} m^2 |m\rangle \tag{7}$$

$$\hat{L}^2 |m\rangle = \hbar^2 m^2 |m\rangle \tag{8}$$

$$\hat{L}_z |m\rangle = \hbar m |m\rangle \tag{9}$$

3 1-D Harmonic Oscillator

Eigenstates:

$$|n\rangle \doteq \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} \frac{1}{\sqrt{2^n n!}} H_n(\xi)\, e^{-\xi^2/2} \tag{10}$$

$$\xi = \sqrt{\frac{m\omega}{\hbar}}\, x \tag{11}$$

$$n = \{0, 1, 2, 3, \dots\} \tag{12}$$

Eigenvalue Equations:

$$\hat{H} |n\rangle = \hbar\omega \left(n + \frac{1}{2}\right) |n\rangle \tag{13}$$

$$\tag{14}$$

4 2-D Particle-in-a-Box

Eigenstates:

$$|mn\rangle \doteq \sqrt{\frac{2}{L_x}}\sqrt{\frac{2}{L_y}}\,\sin\frac{m\pi x}{L_x}\sin\frac{n\pi y}{L_y} \tag{15}$$

$$m = \{1, 2, 3, \dots\} \tag{16}$$

$$n = \{1, 2, 3, \dots\} \tag{17}$$

Eigenvalue Equations:

$$\hat{H}\,|mn\rangle = \frac{\pi^2\hbar^2}{2\mu}\left(\frac{m^2}{L_x^2} + \frac{n^2}{L_y^2}\right)|mn\rangle \tag{18}$$

$$\tag{19}$$

5 Particle-on-a-Sphere

Eigenstates:

$$|\ell m\rangle \doteq Y_\ell^m(\theta, \phi) \tag{20}$$

$$= (-1)^{\frac{m+|m|}{2}}\sqrt{\frac{2\ell+1}{4\pi}\frac{(\ell-m)!}{(\ell+m)!}}\,P_\ell^m(\cos\theta)\,e^{im\phi} \tag{21}$$

$$\ell = \{0, 1, 2, \dots\} \tag{22}$$

$$m = \{\ell, \dots, 0, \dots, -\ell\} \tag{23}$$

Eigenvalue Equations:

$$\hat{H}\,|\ell m\rangle = \frac{\hbar^2}{2I}\,\ell(\ell+1)\,|\ell m\rangle \tag{24}$$

$$\hat{L}^2\,|\ell m\rangle = \hbar^2\,\ell(\ell+1)\,|\ell m\rangle \tag{25}$$

$$\hat{L}_z\,|\ell m\rangle = \hbar\,m\,|\ell m\rangle \tag{26}$$

6 3-D Particle-in-a-Box

Eigenstates:

$$|mnp\rangle \doteq \sqrt{\frac{2}{L_x}}\sqrt{\frac{2}{L_y}}\sqrt{\frac{2}{L_z}}\,\sin\frac{m\pi x}{L_x}\sin\frac{n\pi y}{L_y}\sin\frac{p\pi z}{L_z} \tag{27}$$

$$m = \{1, 2, 3, \dots\} \tag{28}$$

$$n = \{1, 2, 3, \dots\} \tag{29}$$

$$p = \{1, 2, 3, \dots\} \tag{30}$$

Eigenvalue Equations:

$$\hat{H}\,|mnp\rangle = \frac{\pi^2\hbar^2}{2\mu}\left(\frac{m^2}{L_x^2}+\frac{n^2}{L_y^2}+\frac{p^2}{L_z^2}\right)|mnp\rangle \tag{31}$$

$$\tag{32}$$

7 Hydrogen Atom

Eigenstates:

$$|n\ell m\rangle \doteq R_{n\ell}(r)\,Y_\ell^m(\theta,\phi) \tag{33}$$

$$= -\sqrt{\left(\frac{2Z}{na_0}\right)^3\frac{(n-\ell-1)!}{2n[(n+\ell)!]^3}}\left(\frac{2\rho}{n}\right)^\ell e^{-\frac{\rho}{n}}\,L_{n+\ell}^{2\ell+1}(\tfrac{2\rho}{n})(-1)^{\frac{m+|m|}{2}}\sqrt{\frac{2\ell+1}{4\pi}\frac{(\ell-m)!}{(\ell+m)!}}\,P_\ell^m(\cos\theta)\,e^{im\phi} \tag{34}$$

$$\rho = \frac{Zr}{a_0} \tag{35}$$

$$n = \{1,2,3,\dots\} \tag{36}$$

$$\ell = \{0,1,2,\dots,n-1\} \tag{37}$$

$$m = \{\ell,\dots,0,\dots,-\ell\} \tag{38}$$

Eigenvalue Equations:

$$\hat{H}\,|n\ell m\rangle = -\frac{1}{2}\left(\frac{Ze^2}{4\pi\epsilon_0}\right)^2\frac{\mu}{\hbar^2}\frac{1}{n^2}\,|n\ell m\rangle \tag{39}$$

$$= -13.6\text{eV}\,\frac{1}{n^2}\,|n\ell m\rangle \tag{40}$$

$$\hat{L}^2\,|n\ell m\rangle = \hbar^2\,\ell(\ell+1)\,|n\ell m\rangle \tag{41}$$

$$\hat{L}_z\,|n\ell m\rangle = \hbar\,m\,|n\ell m\rangle \tag{42}$$