Electrostatic Potential from Two Charges

- Find a formula for the electrostatic potential $V(\vec{r})$ that is valid everywhere in space for the following two physical situations:
  - Two charges $+Q$ and $+Q$ placed on a line at $z' = D$ and $z'' = -D$.
  - Two charges $+Q$ and $-Q$ placed on a line at $z' = D$ and $z'' = -D$, respectively.

  Solution
  - Two charges $+Q$ and $+Q$ placed on a line at $z' = D$ and $z'' = -D$.

$$V(x,y,z) = \frac{1}{4\pi\epsilon_0}\left(\frac{Q}{\sqrt{x^2+y^2+(z-D)^2}} + \frac{Q}{\sqrt{x^2+y^2+(z+D)^2}}\right)$$

  - Two charges $+Q$ and $-Q$ placed on a line at $z' = D$ and $z'' = -D$.

$$V(x,y,z) = \frac{1}{4\pi\epsilon_0}\left(\frac{Q}{\sqrt{x^2+y^2+(z-D)^2}} - \frac{Q}{\sqrt{x^2+y^2+(z+D)^2}}\right)$$

- Simplify your formulas in the limiting cases of:
  - the x-axis

    Solution
    * Two charges $+Q$ and $+Q$ placed on a line at $z' = D$ and $z'' = -D$.

$$V(x,0,0) = \frac{1}{4\pi\epsilon_0}\left(\frac{2Q}{\sqrt{x^2+D^2}}\right)$$

    * Two charges $+Q$ and $-Q$ placed on a line at $z' = D$ and $z'' = -D$.

$$V(x,0,0) = 0$$

  - the z-axis

    Solution
    * Two charges $+Q$ and $+Q$ placed on a line at $z' = D$ and $z'' = -D$.

$$V(0,0,z) = \frac{1}{4\pi\epsilon_0}\left(\frac{Q}{\sqrt{(z-D)^2}} + \frac{Q}{\sqrt{(z+D)^2}}\right) \tag{1}$$

$$= \frac{1}{4\pi\epsilon_0}\left(\frac{Q}{|z-D|} + \frac{Q}{|z+D|}\right) \tag{2}$$

    * Two charges $+Q$ and $-Q$ placed on a line at $z' = D$ and $z'' = -D$.

$$V(0,0,z) = \frac{1}{4\pi\epsilon_0}\left(\frac{Q}{\sqrt{(z-D)^2}} - \frac{Q}{\sqrt{(z+D)^2}}\right) \tag{3}$$

$$= \frac{1}{4\pi\epsilon_0}\left(\frac{Q}{|z-D|} - \frac{Q}{|z+D|}\right) \tag{4}$$

- Discuss the relationship between the symmetries of the physical situations and the symmetries of the functions in these limiting cases.

Solution

- Two charges $+Q$ and $+Q$ placed on a line at $z' = D$ and $z'' = -D$. This charge distribution is symmetric across both the x- and z-axes. The function

$$V(x,0,0) = \frac{1}{4\pi\epsilon_0}\left(\frac{2Q}{\sqrt{x^2+D^2}}\right)$$

is symmetric under the interchange $x \longrightarrow -x$; it is an even function of x. And the function

$$V(0,0,z) = \frac{1}{4\pi\epsilon_0}\left(\frac{Q}{|z-D|}+\frac{Q}{|z+D|}\right) \tag{5}$$

is symmetric under the interchange $z \longrightarrow -z$; it is an even function of z.

- Two charges $+Q$ and $-Q$ placed on a line at $z' = D$ and $z'' = -D$. This charge distribution is symmetric across the z-axis, but antisymmetric across the x-axis. The function

$$V(x,0,0) = 0$$

is both symmetric and antisymmetric under the interchange $x \longrightarrow -x$; the function 0 is both even and odd. And the function

$$V(0,0,z) = \frac{1}{4\pi\epsilon_0}\left(\frac{Q}{|z-D|}-\frac{Q}{|z+D|}\right) \tag{6}$$

is antisymmetric under the interchange of $z \longrightarrow -z$; it is an odd function of z.