

For $\ell = 1$, the operators that measure the three components of angular momentum in matrix notation are given by:

$$L_x = \frac{\hbar}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad (1)$$

$$L_y = \frac{\hbar}{\sqrt{2}} \begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix} \quad (2)$$

$$L_z = \hbar \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix} \quad (3)$$

Show that:

1. Find the commutator of L_x and L_y .
2. Find the matrix representation of $L^2 = L_x^2 + L_y^2 + L_z^2$.
3. Find the matrix representations of the raising and lowering operators $L_{\pm} = L_x \pm iL_y$.

Solution Notice that L_{\pm} are NOT Hermitian and therefore cannot represent observables. They are used as a tool to build one quantum state from another.

4. Show that $[L_z, L_{\pm}] = \lambda L_{\pm}$. Find λ . Interpret this expression as an eigenvalue equation. What is the operator?
5. Let L_+ act on the following three states given in matrix representation.

$$|1, 1\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad |1, 0\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad |1, -1\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \quad (4)$$

Why is L_+ called a “raising operator”?