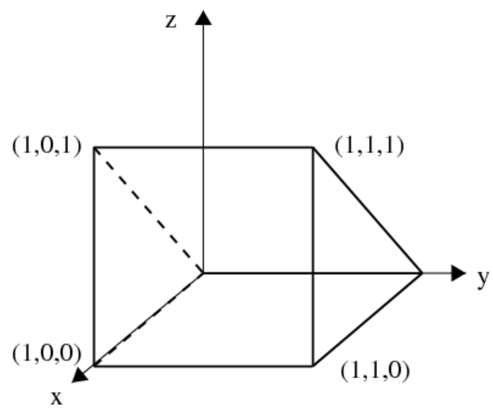
1 Divergence through a Prism

Consider the vector field $\vec{F} = (x+2)\hat{x} + (z+2)\hat{z}$.

(a) Calculate the divergence of $\vec{F}$.

(b) In which direction does the vector field $\vec{F}$ point on the plane $z = x$? What is the value of $\vec{F} \cdot \hat{n}$ on this plane where $\hat{n}$ is the unit normal to the plane?

(c) Verify the divergence theorem for this vector field where the volume involved is drawn below. ("Verify" means calculate both sides of the divergence theorem, separately, for this example and show that they are the same.)



2 Total Current, Circular Cross Section

A current I flows down a cylindrical wire of radius R.

(a) If it is uniformly distributed over the surface, give a formula for the surface current density $\vec{K}$.

(b) If it is distributed in such a way that the volume current density, $|\vec{J}|$, is inversely proportional to the distance from the axis, give a formula for $\vec{J}$.

3 Current in a Wire

The current density in a cylindrical wire of radius R is given by $\vec{J}(\vec{r}) = \alpha s^3 \cos^2\phi\,\hat{z}$. Find the total current in the wire.

4 Differential Form of Gauss's Law

For an infinitesimally thin cylindrical shell of radius b with uniform surface charge density σ, the electric field is zero for $s < b$ and $\vec{E} = \frac{\sigma b}{\epsilon_0 s}\,\hat{s}$ for $s > b$. Use the differential form of Gauss' Law to find the charge density everywhere in space.

5 Square Sheet of Current

Consider a point a distance z above the center of an infinitesimally thin, square sheet of current. The current is parallel to one of the square sides. (Since the current cannot just begin and end in the middle of nowhere, this current is just the building block for some larger current.)

(a) Use the Biot-Savart Law to find the magnetic field at the point z. You may use any symmetry arguments you like, but do **not** use Ampere's Law.

Note: if you choose to use Mathematica or Maple to evaluate the integral, it may take you into complex number land, even though the integral is clearly real. To address this issue, you should be explicit about what assumptions you want the program to make ("Assume" in Maple and "Assumptions" in Mathematica)

(b) Consider your previous answer in the limit that the square becomes infinitely large.

(c) Discuss your answer in the light of the magnetic field above an infinite sheet of current as found using Ampere's Law.

The magnetic field above infinite sheet of current found using Ampere's Law is the same as part b:

$$\vec{B}(0,0,z) = -\frac{\mu_0 K_0}{2}\,\hat{y} \tag{1}$$

Using Ampere's Law is, of course, significantly easier, but it cannot be used in part a.