Submit these problems on Gradescope by 3 pm on Thursday, 10 October.

1 Two Charges

Sketch the equipotentials for two equal positive charges Q separated by a distance D. Make sure to state the reasons for the main features of your sketch. You may use technology to check your answer, but you still need to state reasons for the features. Note: It is an important convention that the VALUES of contour lines should be equally spaced.

2 Potential vs. Potential Energy

In this course, two of the primary examples we will be using are the potential due to gravity and the potential due to an electric charge. Both of these potentials vary like $\frac{1}{r}$, so they will have many, many similarities. Most of the calculations we do for the one case will be true for the other. But there are some extremely important differences:

(a) Find the value of the electrostatic potential energy of a system consisting of a hydrogen nucleus and an electron separated by the Bohr radius. Find the value of the gravitational potential energy of these same two particles at the Bohr radius. Use the same system of units in both cases. Compare and the contrast the two answers.

(b) Find the value of the electrostatic potential due to the nucleus of a hydrogen atom at the Bohr radius. Find the gravitational potential due to the nucleus at the same radius. Use the same system of units in both cases. Compare and contrast the two answers.

(c) Briefly discuss at least one other fundamental difference between electromagnetic and gravitational systems. Hint: Why are we bound to the earth gravitationally, but not electromagnetically?

3 Series Convergence

Recall that, if you take an infinite number of terms, the power series for $\sin z$ and the function itself $f(z) = \sin z$ are equivalent representations of the same thing for all real numbers z, (in fact, for all complex numbers z). This is what it means for the power series to "converge" for all z.

Not all power series converge for all values of the argument of the function. More commonly, a power series is only a valid, equivalent representation of a function for some more restricted values of z, **even if you keep an infinite number of terms**. The technical name for this idea is convergence–the series only "converges" to the value of the function on some restricted domain, called the "interval" or "region of convergence."

(a) Find the power series for the function $f(z) = \frac{1}{1+z^2}$ expanded around $z = 0$..

(b) Using the *Geogebra* applet from class as a model, or some other computer algebra system like Mathematica or Maple, plot both the original function and the power series to explore the convergence of this series. Where does your series for this new function converge? Can you tell anything about the region of convergence from the graphs of the various approximations?

Print your plot and write a brief description (a sentence or two) of the region of convergence.

You may need to include a lot of terms to see the effect of the region of convergence. You may also need to play with the values of z that you plot. Keep adding terms until you see a really strong effect!

Note: As a matter of professional etiquette (or in some cases, as a legal copyright requirement), if you use or modify a computer program written by someone else, you should always acknowledge that fact briefly in whatever you write up. Say something like: "This calculation was based on a (*name of software package*) program titled (*title*) originally written by (*author*) copyright (*copyright date*)."