

1 Helium heat capacity

In class, we assumed that all monatomic gases have 3 degrees of freedom ($f = 3$). In this question, we explore the possibility that a monatomic gas might have additional degrees of freedom due to the electrons orbiting the nucleus, or the rotation of the nucleus. To answer this question, you will need to use the equipartition theorem and understand how quantized energy levels affect the application of the equipartition theorem.

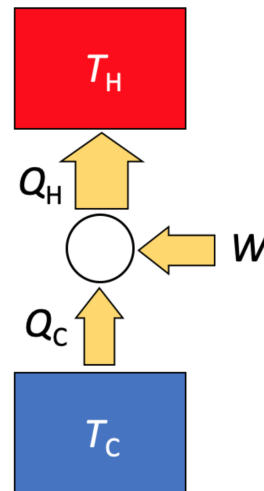
(a) An atom of helium can store energy by bumping its electron from its lowest orbital energy level to a higher orbital energy level. Moving an electron from the lowest state to the first excited state would store an energy of 24.6 eV (24.6 electron-volts). Give a quantitative explanation (i.e. by comparing quantities) that shows we can ignore this energy storage mode when calculating the heat capacity of helium gas at ordinary temperatures.

(b) The helium-4 nucleus can be modelled as a solid spherical object with mass m , radius r , and moment of inertia $I = (2/5)mr^2$. If the nucleus starts to rotate, it would have rotational kinetic energy $K_{\text{rotation}} = L^2/(2I)$, where L is the angular momentum. Usually the helium-4 nucleus has $L = 0$, however, it can be excited to a non-zero angular momentum state with $L \approx \hbar$, or $2\hbar$, or $3\hbar$, etc. (L is quantized). Give a quantitative explanation that shows we can ignore this energy storage mode when calculating the heat capacity of helium gas at ordinary temperatures.

2 Heat Pump

The diagram shows a machine (the white circle) that moves energy from a cold reservoir to a hot reservoir. We will consider whether a machine like this is useful for heating a family home in the winter when the temperature inside the family home is T_H , and the temperature outside the house is T_C . To quantify the performance of this machine, I'm interested in the ratio Q_H/W , where Q_H is the heat energy entering the house, and W is the net energy input in the form of work. (W is the energy I need to buy from the electricity company to run an electric motor). Starting from the 1st and 2nd laws of thermodynamics, find the maximum possible value of Q_H/W . This maximum value of Q_H/W will depend solely on the ratio of temperatures T_H and T_C .

Sensemaking: Choose realistic values of T_H and T_C to describe a family home on a snowy day. Based on your temperature estimates, what is the maximum possible value of Q_H/W ?



3 Photons Absorbed and Reradiated by Earth Estimation

The energy of a single photon (a particle of light) is related to its wavelength:

$$E_{\text{photon}} = \frac{hc}{\lambda}$$

where h is Planck's constant, c is the speed of light, and λ is the wavelength of light.

We've previously talked about how the Earth can function perfectly fine when all the energy we receive from the sun as photons centered in the visible spectrum is reradiated by the Earth as photons centered in the infrared spectrum.

Use a coarse-grained model that:

- all the energy we receive from the Sun is carried by yellow-green photons ($\lambda = 560$ nanometers - the peak of the suns electromagnetic spectrum) and that
- all this energy is re-radiated by Earth as infrared photons ($\lambda = 10$. micrometers, the peak of the electromagnetic spectrum)

to estimate the ratio of the number of photons emitted by Earth to the number of photon incident on the Earth.

4 Standing Waves

None A standing wave is produced with two identical traveling waves move past each other traveling in opposite directions.

- Convince yourself that a standing wave

$$y_S(x, t) = 2 \sin[(\pi \text{ m}^{-1})x] \cos[(\pi \text{ s}^{-1})t]$$

is by a superposition of waves:

$$y(x, t) = \sin[(\pi \text{ m}^{-1})x \pm (\pi \text{ s}^{-1})t]$$

You can do this through the use of trig identities or by graphing the different functions and observing the shapes (turn in screen shots if you go this route).

- Show that the standing wave equation solves the wave equation for a wave on a string.