

1 Bead on a Spinning Wire Hoop

(modified from Taylor Ex. 7.6)

A small bead of mass m is threaded on a frictionless circular wire hoop of radius R . The hoop lies in a vertical plane, which is forced to rotate about the hoop's vertical diameter with constant angular velocity $\dot{\phi} = \omega$, as shown in Figure 7.9. The bead's position on the hoop is specified by the angle θ measured up from vertical.

- Write down the Lagrangian for the system in terms of the generalized coordinate θ and find $\ddot{\theta}$. Discuss at least three strategies for making sense of your answer.
- Find the angles for which $\ddot{\theta} = 0$ - these are equilibrium angles. Show these locations on a sketch of the hoop. Use at least three strategies for making sense of your answer. Include and discuss a plot the equilibrium angles vs. ω

2 A Ball Confined to the Surface of a Sphere in Near-Earth Gravity

A ball with mass m is confined to move on the surface of a sphere with radius $r = R$. A convenient choice of coordinates is spherical, r, θ, ϕ , with the polar axis pointing straight down. (Remember: in physics, ϕ is the azimuthal angle in the xy -plane and θ is the angle with the z -axis.)

- Find the equations of motion using a Lagrangian approach. Use at least three sense-making strategies to evaluate each equation.
- Explain what the ϕ equation indicates about the z -component of angular momentum.
- Discuss the specific special case that $\phi = \text{constant}$. What does the equation of motion for θ indicate?