1   Use What You Know on the Helix
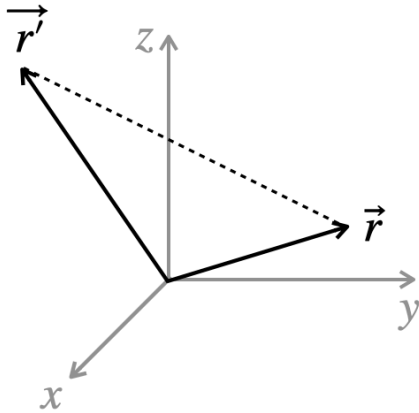
(a) Evaluate $\vec{dr}$ along the helical path $z = 7\phi$.

(b) Evaluate $|d\vec{r}|$ along the helical path $z = 7\phi$.

2   Distance Formula in Curvilinear Coordinates

Review from the Theoretical Mechanics Paradigm

The distance $|\vec{r} - \vec{r}\,'|$ between the point $\vec{r}$ and the point $\vec{r}\,'$ is a coordinate-independent, physical and geometric quantity. But, in practice, you will need to know how to express this quantity in different coordinate systems.

(a) Find the distance $|\vec{r} - \vec{r}\,'|$ between the point $\vec{r}$ and the point $\vec{r}\,'$ in rectangular coordinates.

(b) Show that this same distance written in cylindrical coordinates is:

$$|\vec{r} - \vec{r}\,'| = \sqrt{s^2 + s\,'^2 - 2ss\,'\cos(\phi - \phi\,') + (z - z\,')^2}$$

Hint: You may want to use the textbook: GMM: Change of Coordinates

(c) Show that this same distance written in spherical coordinates is:

$$|\vec{r} - \vec{r}\,'| = \sqrt{r'^2 + r^{\,2} - 2rr\,'\left[\sin\theta\sin\theta\,'\cos(\phi - \phi\,') + \cos\theta\cos\theta\,'\right]}$$

Hint: You may want to use the textbook: GMM: Change of Coordinates

(d) Now assume that $\vec{r}\,'$ and $\vec{r}$ are in the x-y plane. Simplify the previous two formulas.