1 Find Area/Volume from the Vector Differential

(8 pts total)

Start with $d\vec{r}$ in rectangular, cylindrical, and spherical coordinates. Use these expressions to write the scalar area elements dA (for different coordinate equals constant surfaces) and the volume element $d\tau$. It might help you to think of the following surfaces: The various sides of a rectangular box, a finite cylinder with a top and a bottom, a half cylinder, and a hemisphere with both a curved and a flat side, and a cone.

(a) Rectangular:

$$dA = \tag{1}$$
$$d\tau = \tag{2}$$

(b) Cylindrical:

$$dA = \tag{3}$$
$$d\tau = \tag{4}$$

(c) Spherical:

$$dA = \tag{5}$$
$$d\tau = \tag{6}$$

2 Memorize the Vector Differential

(0 pts) Nothing to turn in.

Write $\vec{dr}$ in rectangular, cylindrical, and spherical coordinates.

(a) Rectangular:

$$\vec{dr} = \tag{7}$$

(b) Cylindrical:

$$\vec{dr} = \tag{8}$$

(c) Spherical:

$$\vec{dr} = \tag{9}$$

3 Charge on a Spiral

(8 pts)

A charged spiral in the x, y-plane has 6 turns from the origin out to a maximum radius R , with ϕ increasing proportionally to the distance from the center of the spiral. Charge is distributed on the spiral so that the charge density increases linearly as the radial distance from the center increases. At the center of the spiral the linear charge density is $0\ \frac{\mathrm{C}}{\mathrm{m}}$. At the end of the spiral, the linear charge density is $13\ \frac{\mathrm{C}}{\mathrm{m}}$. What is the total charge on the spiral?

4 Cone Surface

(8 pts)

(a) Find dA on the surface of an (open) cone in both cylindrical and spherical coordinates. Hint: Be smart about how you coordinatize the cone.

(b) Using integration, find the surface area of an (open) cone with height H and radius R. Do this problem in both cylindrical and spherical coordinates.