1 Mass Density

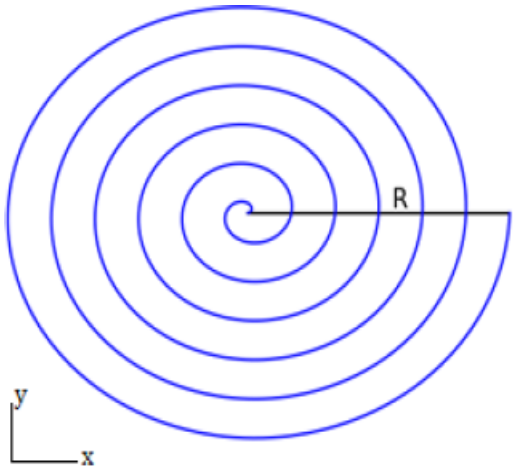
Consider a rod of length L lying on the z-axis. Find an algebraic expression for the mass density of the rod if the mass density at $z = 0$ is λ_0 and at $z = L$ is $7\lambda_0$ and you know that the mass density increases:

(a) with the square of the distance along the rod;

(b) exponentially.

2 Charge on a Spiral

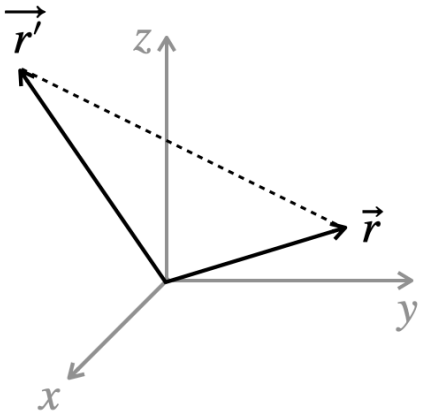
A charged spiral in the x, y-plane has 6 turns starting at the origin out to a maximum radius R. The radial distance from the center to a point on the spiral is proportional to ϕ. The charge density on the spiral increases linearly as the radial distance from the center increases. At the center of the spiral the linear charge density is 0 $\frac{\text{C}}{\text{m}}$. At the end of the spiral, the linear charge density is 13 $\frac{\text{C}}{\text{m}}$. What is the total charge on the spiral?



3 Distance Formula in Curvilinear Coordinates

The distance $|\vec{r} - \vec{r}\,'|$ between the point $\vec{r}$ and the point $\vec{r}'$ is a coordinate-independent, physical and geometric quantity. But, in practice, you will need to know how to express this quantity in different coordinate systems.

(a) Find the distance $|\vec{r} - \vec{r}\,'|$ between the point $\vec{r}$ and the point $\vec{r}'$ in rectangular coordinates.



(b) Show that this same distance written in cylindrical coordinates is:

$$|\vec{r} - \vec{r}\,'| = \sqrt{s^2 + s'^2 - 2ss'\cos(\phi - \phi') + (z - z')^2}$$

Hint: You may want to use the textbook: GMM: Change of Coordinates

(c) Show that this same distance written in spherical coordinates is:

$$|\vec{r} - \vec{r}\,'| = \sqrt{r'^2 + r^2 - 2rr'\left[\sin\theta\sin\theta'\cos(\phi - \phi') + \cos\theta\cos\theta'\right]}$$

Hint: You may want to use the textbook: GMM: Change of Coordinates

(d) Now assume that $\vec{r}\,'$ and $\vec{r}$ are in the x-y plane. Simplify the previous two formulas.