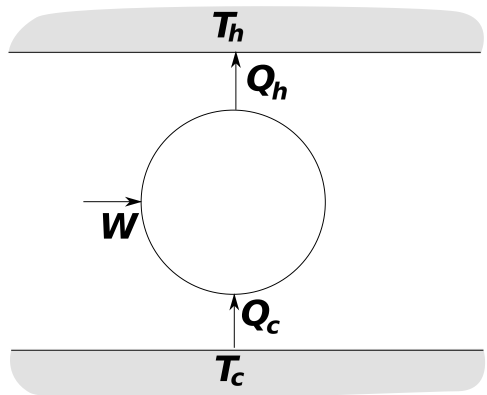
1 Heat Pump

You already read about parts a and b, so you can reference the book on them; focus your answers here on parts c and d.



A heat pump is a refrigerator (or air conditioner) run backwards, so that it cools the outside air (or ground) and warms your house. We will call Q_h the amount of heat delivered to your home, and W the amount of electrical energy used by the pump.

(a) Define a coefficient of performance γ for a heat pump, which (like the efficiency of a heat engine) is the ratio of "what you get out" to "what you put in."

(b) Use the second law of thermodynamics to find an equation for the coefficient of performance of an ideal (reversible) heat pump, when the temperature *inside* the house is T_h and the temperature *outside* the house is T_c. What is the coefficent of performance in the limit as $T_c \ll T_h$?

(c) Discuss your result in the limit where the indoor and outdoor temperatures are close, i.e. $T_h - T_c \ll T_h$. Does it make sense?

(d) What is the ideal coefficient of performance of a heat pump when the indoor temperature is 70°F and the outdoor temperature is 50°F? How does it change when the outdoor temperature drops to 30°F?

2 Power Plant on a River

At a power plant that produces 1 GW (10^9watts) of electricity, the steam turbines take in steam at a temperature of 500^oC, and the waste energy is expelled into the environment at 20^oC.

(a) What is the maximum possible efficiency of this plant?

(b) Suppose you arrange the power plant to expel its waste energy into a chilly mountain river at 15^oC. Roughly how much money can you make in a year by installing your improved hardware, if you sell the additional electricity for 10 cents per kilowatt-hour?

(c) At what rate will the plant expel waste energy into this river?

(d) Assume the river's flow rate is 100 m^3/s. By how much will the temperature of the river increase?

(e) To avoid this "thermal pollution" of the river the plant could instead be cooled by evaporation of river water. This is more expensive, but it is environmentally preferable. At what rate must the water evaporate? What fraction of the river must be evaporated?

3 Bottle in a Bottle Part 2

None

Consider the bottle-in-a-bottle problem in a previous problem set, summarized here. A small bottle of helium is placed inside a large bottle, which otherwise contains vacuum. The inner bottle contains a slow leak, so that the helium leaks into the outer bottle. The inner bottle contains one tenth the volume of the outer bottle. The outer bottle is insulated.

The volume of the small bottle is 0.001 m^3 and the volume of the big bottle is 0.01 m^3. The initial state of the gas in the small bottle was $p = 106$ Pa and its temperature $T = 300$ K. Approximate the helium gas as an ideal gas of equations of state $pV = Nk_BT$ and $U = \frac{3}{2}Nk_BT$.

(a) How many molecules of gas are initially in the small bottle? What is the final temperature of the gas after the pressures have equalized?

(b) Compute the change of entropy ΔS between the initial state (gas in the small bottle) and the final state (gas in both bottles, pressures equalized). Do not use the Sackur-Tetrode equation, use an alternative method.

(c) Discuss your results.

4 Adiabatic Ideal Gas

None

Consider the adiabatic expansion of a simple ideal gas (adiabatic means that no energy is transfered by heating). The internal energy is given by

$$U = C_vT \tag{1}$$

where you may take C_v to be a constant—although for a polyatomic gas such as oxygen or nitrogen, it is temperature-dependent. The ideal gas law

$$pV = Nk_BT \tag{2}$$

determines the relationship between p, V and T. You may take the number of molecules N to be constant.

(a) Use the first law to relate the inexact differential for work to the exact differential dT for an adiabatic process.

(b) Find the total differential dT where T is a function $T(p, V)$.

(c) In the previous two sections, we found two formulas involving dT. Use the additional definition of work $đW = -pdV$ to solve for the relationship between p, dp, V and dV for an adiabatic process.

(d) Integrate the above differential equation to find a relationship between the initial and final pressure and volume for an adiabatic process.

5 Adiabatic Compression

None

A diesel engine requires no spark plug. Rather, the air in the cylinder is compressed so highly that the fuel ignites spontaneously when sprayed into the cylinder.

In this problem, you may treat air as an ideal gas, which satisfies the equation $pV = Nk_BT$. You may also use the property of an ideal gas that the internal energy depends only on the temperature T, i.e. the internal energy does not change for an isothermal process. For air at the relevant range of temperatures the heat capacity at fixed volume is given by $C_V = \frac{5}{2}Nk_B$, which means the internal energy is given by $U = \frac{5}{2}Nk_BT$.

Note: Looking up the formula in a textbook is *not* considered a solution at this level. Use only the equations given, fundamental laws of physics, and results you might have already derived from the same set of equations in other homework questions.

(a) If the air is initially at room temperature (taken as 20^oC) and is then compressed adiabatically to $\frac{1}{15}$ of the original volume, what final temperature is attained (before fuel injection)?

(b) By what factor does the pressure increase (before fuel injection)?