



1 A Physical System that Oscillates

A mass m is strung on a very light wire equidistant between two anchor points separated by a distance $2L$. When the mass is displaced laterally (*i.e.* plucked) in the x -direction (here vertically), there is constant tension T on the wire that produces a restoring force.

- (a) What is the resulting natural frequency ω_0 of the oscillating mass? Assume tension drives the oscillation (*i.e.* no gravity) and assume the angle the string makes with the posts is small.
- Note:** ω_0 must be expressed in terms of the physical parameters given in the problem (T, L, m).
- (b) Write a generic expression for displacement from equilibrium in any of the standard forms, $ABCD$.

2 Phase lead/lag

- (a) Sketch on one plot, two oscillations (#1 and #2) of the same period (1 s), where oscillation #2 has an amplitude that is twice that of oscillation #1 and lags oscillation #1 by $\frac{\pi}{4}$.
- (b) Sketch on one plot, the charge

$$q(t) = Q \cos(\omega_0 t)$$

and the current

$$I(t) = \dot{q}(t)$$

and say which one leads and by what phase. Give a verbal explanation.

Note: It is good to object to plotting 2 quantities with different dimensions on the same vertical axis! But it is okay if you think of the plot as having a left vertical axis for one quantity and a right axis for the other. The important thing is to get the horizontal time axis to line up for both oscillations.

3 Damped Oscillator

An undamped oscillator has a period $T_1 = 1.000$ s, that increases to $T_2 = 1.001$ s when damping is added.

- (a) What is the damping factor β ?
- (b) By what factor does the oscillation amplitude decrease after 10 cycles?

4 Underdamped LRC Oscillator

Consider an oscillator with capacitance $C = 18 \text{ nF} = 18 \times 10^{-9} \text{ F}$, a resistor of resistance $R = 50 \Omega$ and an inductor with an inductance $L = 22 \text{ mH} = 0.022 \text{ H}$. Charge the isolated capacitor up and then connect it *in series* to the L and R and allow it to discharge. Assume the starting time is $t = 0$.

- (a) What are the initial conditions for this circuit?
- (b) What is the damping time (time for amplitude to decay to $1/e$ of starting value)?
- (c) By what fraction is the oscillation frequency shifted from the undamped version?
- (d) How many cycles occur within the damping time?
- (e) What is the value of the quality factor (or Q-factor) of the circuit? Look up another system to put this Q-factor in some sort of context.
- (f) How long will it take for the system lose 90% of its energy?
- (g) Plot the charge, current, and current derivative (\dot{I}) in the circuit as functions of time for enough cycles to show the damping. Use Mathematica, Python, or the software of your choice.