1 Eigen Practice

None

Find the eigenvectors and eigenvalues of the matrices from the Linear Transformations small group activity from Tuesday's class. Keep working until you are fluent. Make up some 2×2 and 3×3 matrices of your own if you need more practice.

2 Diagonalization

None

(a) Let

$$|\alpha\rangle \doteq \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix} \qquad \text{and} \qquad |\beta\rangle \doteq \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix}$$

Show that $|\alpha\rangle$ and $|\beta\rangle$ are orthonormal. (If a pair of vectors is orthonormal, that suggests that they might make a good basis.)

(b) Consider the matrix

$$C \doteq \begin{pmatrix}3 & 1\\1 & 3\end{pmatrix}$$

Show that the vectors $|\alpha\rangle$ and $|\beta\rangle$ are eigenvectors of C and find the eigenvalues. (Note that showing something is an eigenvector of an operator is far easier than finding the eigenvectors if you don't know them!)

(c) A operator is always represented by a diagonal matrix if it is written in terms of the basis of its own eigenvectors. What does this mean? Find the matrix elements for a new matrix E that corresponds to C expanded in the basis of its eigenvectors, i.e. calculate $\langle\alpha|C|\alpha\rangle$, $\langle\alpha|C|\beta\rangle$, $\langle\beta|C|\alpha\rangle$ and $\langle\beta|C|\beta\rangle$ and arrange them into a sensible matrix E. Explain why you arranged the matrix elements in the order that you did.

(d) Find the determinants of C and E. How do these determinants compare to the eigenvalues of these matrices?

3 Eigen Spin Challenge

None

Consider the arbitrary Pauli matrix $\sigma_n = \hat{n}\cdot\vec{\sigma}$ where $\hat{n}$ is the unit vector pointing in an arbitrary direction.

(a) Find the eigenvalues and normalized eigenvectors for σ_n. The answer is:

$$\begin{pmatrix}\cos\frac{\theta}{2}e^{-i\phi/2}\\ \sin\frac{\theta}{2}e^{i\phi/2}\end{pmatrix} \begin{pmatrix}-\sin\frac{\theta}{2}e^{-i\phi/2}\\ \cos\frac{\theta}{2}e^{i\phi/2}\end{pmatrix}$$

It is not sufficient to show that this answer is correct by plugging into the eigenvalue equation. Rather, you should do all the steps of finding the eigenvalues and eigenvectors as if you don't know the answer. Hint: $\sin\theta = \sqrt{1-\cos^2\theta}$.

(b) Show that the eigenvectors from part (a) above are orthogonal.

(c) Simplify your results from part (a) above by considering the three separate special cases: $\hat{n} = \hat{\imath}$, $\hat{n} = \hat{\jmath}$, $\hat{n} = \hat{k}$. In this way, find the eigenvectors and eigenvalues of σ_x, σ_y, and σ_z.