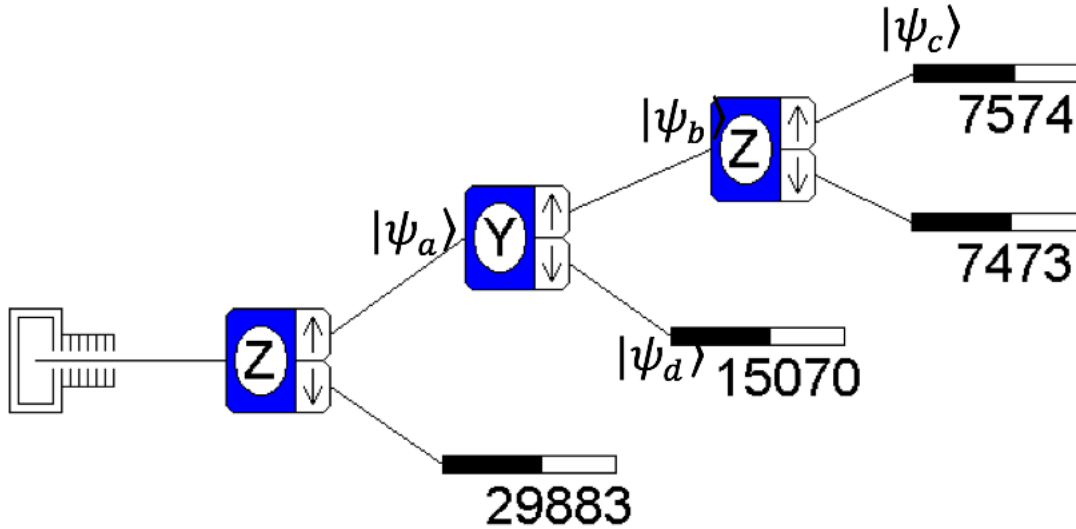


## 1 Chained Stern-Gerlach (Brief)

Consider the Stern-Gerlach set-up shown, with a thermal oven source, and some state vectors associated with various analyzer outputs:



- How many particles were released from the oven?
- Write down Dirac notation expressions for  $|\Psi_a\rangle$ ,  $|\Psi_b\rangle$ ,  $|\Psi_c\rangle$ , and  $|\Psi_d\rangle$  in the  $S_z$  basis (i.e., as linear superpositions of  $|+\rangle_z$  and  $|-\rangle_z$ ).

## 2 Spin One Half Unknowns (Brief)

Using the SPINS simulation for a spin- $\frac{1}{2}$  system (Spins simulation), for the unknown initial state  $|\psi_4\rangle$ , perform measurements of  $S_x$ ,  $S_y$ , and  $S_z$  separately, and for each measurement determine the probability of obtaining each possible measurement value. (We carried out the same procedure for  $|\psi_3\rangle$  in class; you may refer to that example.)

- Use the probabilities you observed from the measurements to express  $|\psi_4\rangle$  as a linear superposition of the  $S_z$ -basis states  $|+\rangle$  and  $|-\rangle$ .
- Articulate a Process:* Write a set of general instructions that would allow another student in next year's class to find an unknown state from probabilities obtained from spin measurements.
- Compare Theory with Experiment:* Design an experiment that will allow you to test whether the state you identified in Question (a) for the unknown state  $|\psi_4\rangle$  is correct. Describe your experiment here, clearly but succinctly, as if you were writing it up for a paper. Do the experiment and discuss your results.

- (d) *Make a Conceptual Connection:* In general, are the probabilities obtained from spin-component measurements along only two spin directions (for example, the  $z$  direction and the  $y$  direction) sufficient to determine a spin- $\frac{1}{2}$  quantum state? Why or why not?

### 3 Spin Calculations (brief)

A set of spin-1/2 particles are identically prepared in the state

$$|\psi_A\rangle = N\left(\sqrt{3}|+\rangle + e^{i\pi/3}|-\rangle\right)$$

- .
- (a) Determine  $N$  so that the state is normalized.
  - (b) What values of the  $z$ -component of spin might you measure and with what probabilities?
  - (c) What values of the  $y$ -component of spin might you measure and with what probabilities?
  - (d) Write this state in the  $S_x$  basis (i.e., as a linear superposition of  $|+\rangle_x$  and  $|-\rangle_x$ ).
  - (e) In what direction would you have to orient a Stern-Gerlach analyzer so that ALL the particles prepared in the state  $|\psi_A\rangle$  would be measured to have a spin component in that direction equal to  $+\hbar/2$ ? Give the direction in spherical coordinates,  $\theta$  and  $\phi$ .