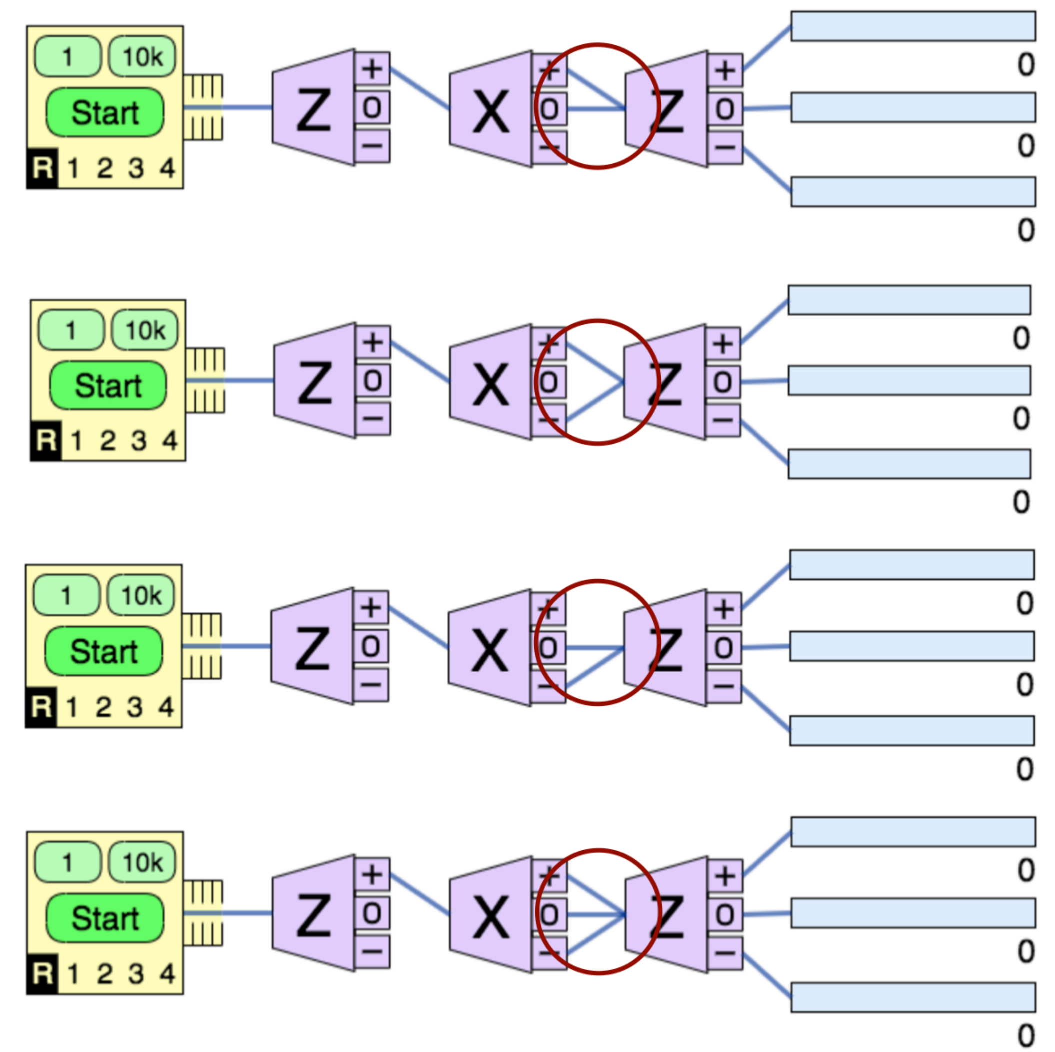
1 Spin One Interferometer Brief

(4, 4, 4 points) Consider a spin 1 interferometer which prepares the state as $|1\rangle$, then sends this state through an S_x apparatus and then an S_z apparatus. For the four possible cases where a pair of beams or all three beams from the S_x Stern-Gernach analyzer are used, calculate the probabilities that a particle entering the last Stern-Gerlach device will be measured to have each possible value of S_z. Compare your theoretical calculations to results of the simulation. Make sure that you explicitly discuss your choice of projection operators.

Note: You do not need to do the first case, as we have done it in class.



2 Spin One Eigenvectors

(2, 4 pts) The operator $\hat{S}_x$ for spin-1 particles, can be written in matrix form in the S_z basis as:

$$\hat{S}_x = \frac{\hbar}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

(a) Find the eigenvalues of this matrix.

(b) Find the eigenvectors corresponding to each eigenvalue.

3 Finding Matrix Elements

(2, 2, 2 pts)

(a) Carry out the following matrix calculations.

$$\begin{pmatrix} 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

and

$$\begin{pmatrix} 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

(b) What matrix multiplication would you do if you wanted the answer to be a_{31}?

(c) In the first question above, the bra/ket representations for the calulations are:

$$\langle 2| A |1\rangle = ? \quad \text{and} \quad \langle 2| A |2\rangle = ?$$

Write the second question in bra/ket notation.

4 Spin Three Halves Operators

(2, 2, 2, 2, 2 points) If a beam of spin-3/2 particles is input to a Stern-Gerlach analyzer, there are four output beams whose deflections are consistent with magnetic moments arising from spin angular momentum components of $\frac{3}{2}\hbar$, $\frac{1}{2}\hbar$, $-\frac{1}{2}\hbar$, and $-\frac{3}{2}\hbar$. For a spin-3/2 system:

(a) Write down the eigenvalue equations for the $\hat{S}_z$ operator.

(b) Write down the matrix representation of the $\hat{S}_z$ eigenstates in the S_z basis.

(c) Write down the matrix representation of the $\hat{S}_z$ operator in the S_z basis.

(d) Write down the eigenvalue equations for the $\hat{S^2}$ operator. (The eigenvalues of the S^2 are $\hbar^2 s(s+1)$, where s is the spin quantum number. $\hat{S^2} = (\hat{S}_x)^2 + (\hat{S}_y)^2 + (\hat{S}_z)^2$, which is proportional to the identify operator. For spin-3/2 system, $s = \frac{3}{2}$)

(e) Write down the matrix representation of the $\hat{S^2}$ operator in the S_z basis. *Check Beasts:* Is your operator proportional to the identity operator?