1 Commute

(2, 2+2, 2+2, 2 points)

Consider a three-dimensional state space. In the basis defined by three orthonormal kets $|1\rangle$, $|2\rangle$, and $|3\rangle$, the operators A and B are represented by:

$$\hat{A} \doteq \begin{pmatrix} a_1 & 0 & 0 \\ 0 & a_2 & 0 \\ 0 & 0 & a_3 \end{pmatrix} \qquad \hat{B} \doteq \begin{pmatrix} b_1 & 0 & 0 \\ 0 & 0 & b_2 \\ 0 & b_2 & 0 \end{pmatrix}$$

where all the matrix elements are real.

(a) Do the operators $\hat{A}$ and $\hat{B}$ commute?

(b) Find the eigenvalues and normalized eigenvectors of $\hat{A}$ and of $\hat{B}$.

(c) Assume the system is initially in the state $|2\rangle$. Then the observable corresponding to the operator $\hat{B}$ is measured. What are the possible measurement values, and what is the probability of obtaining each value? After this measurement, the observable corresponding to the operator A is measured. What are the possible measurement values, and what is the probability of obtaining each value?

(d) *Interpret the Mathematical Model:* How does the commutator $[\hat{A}, \hat{B}]$ found in part (1) help you understand the measurement outcomes and probabilities in part (3)?

2 General State

(2, 2+2+1, 2+2+1 points)

Use a New Representation: Consider a quantum system with an observable A that has three possible measurement results: a_1, a_2, and a_3. States $|a_1\rangle$, $|a_2\rangle$, and $|a_3\rangle$ are eigenstates of the operator $\hat{A}$ corresponding to these possible measurement results.

(a) Using matrix notation, express the states $|a_1\rangle$, $|a_2\rangle$, and $|a_3\rangle$ in the basis formed by these three eigenstates themselves.

(b) The system is prepared in the state:

$$|\psi_b\rangle = N\left(1\,|a_1\rangle - 2\,|a_2\rangle + 5\,|a_3\rangle\right)$$

  (a) Staying in bra-ket notation, find the normalization constant.

  (b) Calculate the probabilities of all possible measurement values when measuring the observable A. *Check "beasts."*

(c) In a different experiment, the system is prepared in the state:

$$|\psi_c\rangle = N\left(2\,|a_1\rangle + 3i\,|a_2\rangle\right)$$

(a) Find the normalization constant and write this state in matrix notation in the basis $\{|a_1\rangle, |a_2\rangle, |a_3\rangle\}$.

(b) Calculate the probabilities of all possible measurement values when measuring the observable A. *Check "beasts".*

3 Spin Uncertainty

(2, 6 points) Consider the state $|-1\rangle_y$ in a spin 1 system.

(a) Discuss the direction of the spin angular momentum for this quantum system.

(b) In the state $|-1\rangle_y$, calculate the expectation values and uncertainties for measurements of S_x, S_y, and S_z.